Intermediary Funding Liquidity and Rehypothecation as a Determinant of Repo Haircuts and Interest Rates*

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Abstract

This paper offers a theory by which dealer banks obtain funding liquidity by serving as intermediaries between hedge funds and cash investors in the markets for repurchase (repo) agreements. The model explains how the demand by dealer banks for funding liquidity determines repo haircuts and repo pricing. A dealer bank obtains liquidity to the extent of the spread between the haircut on its repos with cash investors and the haircut on its reverse repos with hedge funds. Dealer banks optimally choose the extent to which they use this funding mechanism over alternatives such as cash holdings and fire sales of illiquid assets. Rehypothecation and over-collateralization might expose hedge funds to the bankruptcy risk of dealer banks. The model pins down repo haircuts and interest rates jointly. Haircut spreads are low and hedge funds are not exposed to the bankruptcy risk of dealers when liquidity is abundant. When liquidity is relatively scarce, haircut spreads are high and hedge funds are exposed to the bankruptcy risk of dealers. The model highlights the volume of lending by cash investors and dealer balance sheets as key determinants of haircut spreads. The model yields further testable implications supported by the data.

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1 Introduction

This paper offers a theory by which dealer banks obtain funding liquidity by serving as intermediaries between hedge funds and cash investors[1] in the markets for repurchase (repo) agreements.[2] The model explains how the demand by dealer banks for funding liquidity determines repo haircuts and repo pricing.

Dealer banks finance their securities in large part by repo agreements with cash investors. Hedge funds obtain financing for their investments from the prime brokerage divisions of dealer banks. This financing is substantially in the form of bilateral repos and margin loans. Both of these forms of financing are, in effect, secured loans.[3] In providing this financing, the prime broker is usually given the right to repledge collateral obtained from hedge funds to cash investors.

Figure 1 illustrates the mechanism. This form of rehypothecation serves two main purposes. First, through their back-to-back repos, dealer banks can intermediate the provision of funding to hedge funds by repledging hedge fund collateral to cash investors. Second, by charging higher haircuts to hedge funds than those of their repos with cash investors, dealer banks obtain extra funding for themselves, to the extent of the difference in haircuts. In this paper, I show how and when dealer banks use this funding mechanism over alternatives such as cash holdings and fire sales of illiquid assets.

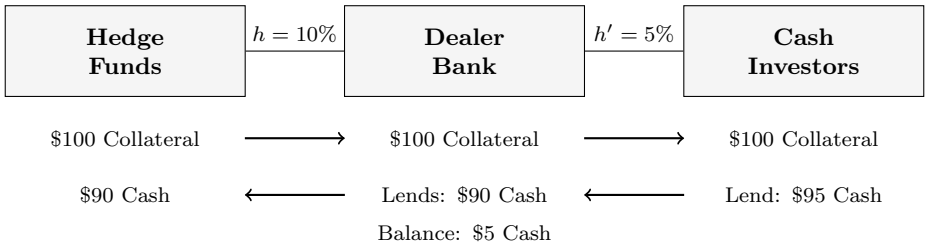

Figure 1: Intermediation and liquidity creation by rehypothecation. By repledging hedge funds' collateral, a dealer bank intermediates cash between cash investors and hedge funds. If the dealer bank charges a higher haircut than it faces, it can obtain liquidity to use for its own purposes. Total liquidity it obtains amounts to the value of the collateral multiplied by the difference in haircuts.

When a dealer bank is given the right of rehypothecation and exercises this

[1] Typical cash investors are money market mutual funds.

[2] Dealer banks and cash investors typically meet in a tri-party repo market, whereas dealer banks and hedge funds meet in a bilateral repo market. For detailed discussion of the market structures, see Copeland, Martin, and Walker [2010], Copeland, Martin, and Walker [2011] and Martin, Skeie, and von Thadden [2014].

[3] Duffie [2010] provides a detailed account of how dealer banks work and how their relationship with cash investors and hedge funds had an impact on their distress during the financial crisis. Duffie [2013] discusses the issues that remain in the "plumbing" of the US financial markets after the financial crisis, including issues in the nexus between dealer banks and cash investors, and dealer banks and hedge funds.

right, the title to the collateral is transferred to cash investors. Moreover, repos are exempt from automatic stay in the event of bankruptcy. Hence, in the event of bankruptcy by a dealer bank, cash investors can liquidate the collateral. Hedge funds would in this case need to present claims for their losses in bankruptcy proceedings. Hedge funds are exposed to ultimate loss to the extent of the haircut on their collateral. For example, suppose a hedge fund borrows \$90 cash from a dealer bank by pledging collateral, with a market value of \$100. The haircut is 10%[4]. For simplicity, suppose the interest rate is zero. Further suppose that the dealer bank then repledges the collateral to a money market mutual fund. If the dealer bank goes bankrupt, then the money market mutual fund will liquidate the collateral. Final leg of the repo will not settle. The dealer bank is unable to return the collateral and hedge funds will not repay. The hedge fund will present a claim for $\$100 - \$90 = \$10$ in the bankruptcy of the dealer, and recover some fraction of this claim, pro rata with unsecured creditors. Higher haircuts thus increase the expected default loss of the hedge fund.

This leads to the main trade-off examined in the paper. Given the exposure that hedge funds face in the event of the bankruptcy of a dealer, they must be given the incentive to participate in a repo agreement. In order to provide incentives to hedge funds to participate in a repo agreement and allow the rehypothecation of the collateral, hedge funds are compensated by lower repo interest rates. Higher haircuts increase over-collateralization. Hence, in a market equilibrium, repos with higher haircuts receive lower repo rates.[5] Dealer banks optimally choose the extent to which they use this funding mechanism over alternatives such as cash holdings and fire sales of illiquid assets. This decision pins down, in the modeled equilibrium, haircut spreads and interest rate spreads between different repo markets. Furthermore, the quality of the collateral determines haircut and interest rate levels in the repo market between dealer banks and cash investors. Hence, haircuts and interest rates in both repo markets are uniquely determined.

Key variables that determine haircut spreads and interest rate spreads are the total amount of funds from cash investors, the cash holdings and short-term debt obligations of dealer banks, as well as the degree of competitiveness in the dealer bank sector[6] and hedge fund characteristics. To clearly demonstrate the results of the mechanism that I propose, I assume that haircuts that dealer banks charge to hedge funds and haircuts that cash investors charge to dealer banks are sufficient to fully secure their lending. Hence, collateral risk elevates haircuts in both markets equally, and has no impact on the haircut spreads.

The main results of the paper are the following: Haircut spreads are low when funding liquidity of dealer banks is abundant. When funding liquidity of dealer banks is scarce, haircut spreads are high. The funding provided by cash investors may dry-up suddenly, as documented and discussed in Krishnamurthy,

[4] $Haircut = \frac{100-90}{100}$

[5] Non-participation of hedge funds could be thought of as a run by hedge funds on collateral which would tighten funding conditions for dealer banks, as argued in Duffie [2010]

[6] This will be defined precisely in the model.

Nagel, and Orlov [2014] and Copeland, Martin, and Walker [2011]. In that case, haircut spreads increase in bilateral repo markets, as documented in Gorton and Metrick [2012] and Copeland, Martin, and Walker [2011]. The model suggests that when the funding liquidity available to dealer banks is abundant, hedge funds are not exposed to the bankruptcy risk of dealer banks. On the other hand, when the funding liquidity available to dealer banks is relatively scarce, haircut spreads are higher and hedge funds are exposed to the bankruptcy risk of dealer banks. Therefore, dealer banks earn lower intermediation profits.

A dealer bank's option to obtain liquidity from a fire sale of assets provides additional insights. When under liquidity stress, dealer banks decide optimally whether to obtain liquidity by fire sales or by haircut spreads. The model suggests that higher haircut spreads are preferable to fire sales when the fire-sale value of illiquid assets is low.[7]

2 Discussion of the Related Literature and Empirical Evidence

Models of haircuts in the literature focus on collateral risk and borrower risk as determinants of haircuts. Geanakoplos [2010] and Simsek [2013] focus on disagreements between borrowers and lenders about the market value of the collateral as a determinant of haircuts and repo interest rates. Gorton and Ordoñez [2014] focus on an information acquisition problem regarding the underlying value of collateral. Martin, Skeie, and von Thadden [2014] propose a model of repo runs, where they also explain the haircut spreads between different repo markets by differences in the microstructure of bilateral and tri-party markets. Infante [2014] also studies repo intermediation of dealer banks with a focus on the bargaining problem between dealer banks and hedge funds. The contribution of this paper is a demonstration of how the terms of dealer repos respond to the incentives of dealer banks to finance themselves through the spread between the haircuts of their reverse repos with prime brokerage clients and their repos with cash investors such as money market mutual funds. In this paper, haircuts and repo rates are pinned down in both markets. Haircut spreads are determined by the volume of lending by cash investors and dealer bank balance sheets. Interest rates are determined by the participation decision of hedge funds. This paper can also provide an explanation for all of the empirical evidence about haircuts and rehypothecation behavior before, during and after 2008 as discussed below.

During the financial crisis, haircuts remained roughly stable in tri-party repo markets, while money market mutual funds and other cash investors reduced lending to dealer banks as documented by Krishnamurthy, Nagel, and Orlov [2014] and Copeland, Martin, and Walker [2011]. This was partly due to the fact that money market mutual funds and other cash investors viewed some

[7] A possible extension of the model can be an analysis of the relationship between funding liquidity and market liquidity as in Brunnermeier and Pedersen [2009].

dealer banks as risky and decided to reduce exposure to them. For example, funding liquidity in the tri-party market for Lehman Brothers dried up prior to its bankruptcy. Lehman Brothers' tri-party repo book declined by $97.8 billion between September 9 and September 16 as documented by Copeland, Martin, and Walker [2010]. The second reason for the reduction of lending by money market mutual funds and other cash investors was that their investors redeemed their shares rapidly. This sharply reduced the maximum amount of funding that they provide, in aggregate, to dealers. The mean daily amount of collateral posted in the tri-party market declined from around $ 2.5 trillion to around $ 1.6 trillion between September 2008 and September 2009[8]. Haircuts increased significantly and became more volatile for similar securities in bilateral repo markets as documented by Gorton and Metrick [2012], Copeland, Martin, and Walker [2011] and CGFS [2010].

The total volume of rehypothecation and the velocity of pledged collateral declined during the crisis. Singh and Aitken [2009], Singh and Aitken [2010] and Singh [2011] provide some evidence on rehypothecation. I provide additional documentation of descriptive evidence from the 10-Q and 10-K SEC filings of major US dealer banks from 2000-2014 in Section 7.

Finally, other related empirical evidence is on the relationship between prime brokers and hedge fund returns. Aragon and Strahan [2012] document data on hedge fund performance and failure rates and find that Lehman clients were affected by Lehman's distress. In a related paper, Klaus and Rzepkowski [2009] document that increases in prime brokers' distress are associated with a significant decline in the performance of their clients. Moreover, hedge funds that rely on multiple prime brokers tend to have higher returns.

3 Model Setup

The model consists of two periods, $t \in \{1, 2\}$. There are three types of agents in the model. The agents are a continuum [0,1] of competitive hedge funds, a continuum [0,1] of competitive cash investors and dealer banks. In this paper, I focus on the problem of a single dealer bank, where the terms of other dealer banks appear as an outside option to hedge funds. All agents are risk neutral and maximize their cash holdings in period 2.

In the model, there are cash, bonds and investment projects of hedge funds and dealer banks. Cash can be invested, while bonds can only be used as collateral to secure loans. All lending in the model is assumed to be backed by collateral and recourse. It means that in case of default, the non-defaulting party has access the balance sheets of the defaulting party. Thus, the non-defaulting party can potentially recover any losses not backed by collateral.

In order to illustrate the mechanism clearly, in the benchmark model I abstract from collateral risk and borrower risk and add it as an extension later on. Bonds are safe. Each bond yields $1 in period 2. There is no market for

[8] See Figure 9

trading bonds in period 1. The only use of bonds in period 1 is as collateral to back cash borrowing.

Each hedge fund $i \in [0,1]$ enters period 1 with b_i bonds. There are $B \equiv \int_0^1 b_i \, di$ bonds in total. Hedge funds have an investment project that converts \$1 in period 1 into $R^H > 1$ dollars in period 2. Hedge funds cannot directly borrow from cash investors in the model. Dealer banks serve as intermediaries between hedge funds and cash investors.[9] Intermediation takes place when the dealer bank repledges hedge funds' collateral to cash investors.

The dealer bank makes take-it-or-leave-it offers to every hedge fund i that specify the haircut, the repo interest rate and the amount of bonds that will be repledged, (h_i, r_i, b_i^r). I assume that hedge funds in this model have a relationship established with the dealer bank that is modeled. Before considering other alternatives, they consider offers made by their dealer bank. Any indifference is resolved by accepting contracts offered by their dealer bank. I assume that dealer banks have all the bargaining power.

If a hedge fund i accepts an offer by a dealer bank, it gives all its bonds to that dealer bank. The dealer bank repledges b_i^r bonds and lends $(1-h_i)b_i^r$ dollars to the hedge fund. The rest of the bonds are kept in custody by the dealer bank in a segregated account that hedge funds can recover in case of bankruptcy.

Each cash investor j has q_j dollars in period 1. There is a total of $\bar{Q}^m \equiv \int_0^1 q_j \, dj$ dollars in this sector. Cash investors need to store their cash holdings and I assume that the only storage technology available to them is reverse repo agreements with dealer banks. In period 1, the total amount of cash that cash investors lend to the dealer bank modeled here, Q^m, is exogenous ($Q_m \leq \bar{Q}_m$). For the rest of the paper, I assume the amount of bonds hedge funds have is greater than the amount of cash that cash investors lend ($B \geq Q_m$). I will discuss the opposite case later. Haircuts in repo agreements between dealer banks and cash investors are zero, since collateral is assumed to be riskless and cash investors are competitive. Cash investors earn the risk free rate, which I assume to be zero.

The dealer bank starts period 1 with total assets, E, denominated in dollars. It has $\alpha^* E$ of total assets that are illiquid. The illiquid assets of the dealer bank yield $R^B > 1$ dollars in period 2, per dollar invested. The rest of its assets, $(1-\alpha^*)E$ is the cash holdings. In the benchmark model, I assume illiquid assets of the dealer bank cannot be liquidated in period 1. I analyze an extension where liquidation is possible.

I assume that there is an exogenous probability, p, that the dealer bank is solvent in period 2. The complementary probability, $1-p$, is the probability that the dealer bank receives a severe adverse shock and goes bankrupt.[10] Due

[9] In the model, I take this intermediation chain as exogenous. In reality, this might be due to an adverse selection problem. An interesting research question would be to understand the underlying frictions that give rise to this intermediation chain.

[10] Bankruptcy can arise either exogenously in period 2 with probability $1-p$ or endogenously in period 1 when expected cash holdings of the dealer bank in period 2 is negative, which will be discussed in Section 4.

to limited liability, the payoff of the dealer bank is zero in bankruptcy. For simplicity, I assume hedge funds cannot recover their losses in period 2 in the event of bankruptcy which can be motivated by the high costs of recovering losses in the bankruptcy proceedings.

In period 1, the dealer bank needs to manage liquidity to maximize expected cash holdings in period 2. In period 1, the dealer bank has a short term debt λE, which matures in period 1. In the benchmark model, the dealer bank chooses between two alternatives to pay back the short term creditors. First alternative is to use the existing cash holdings, $(1 - \alpha^*)E$ in period 1. Second alternative is to obtain liquidity by haircut spreads. Haircut spreads that the dealer bank need depend on the amount of liquidity provided by cash investors and the balance sheet of the dealer, as explained in the two examples below.

Suppose the dealer bank needs \$20 to roll over maturing short-term debt in period 1 and suppose it has no cash holdings. Suppose cash investors have \$100 in cash to lend and the collateral is riskless. By repledging bonds valued at \$100 and charging hedge funds a haircut (spread) of 20%, the dealer bank could generate \$20 for itself.

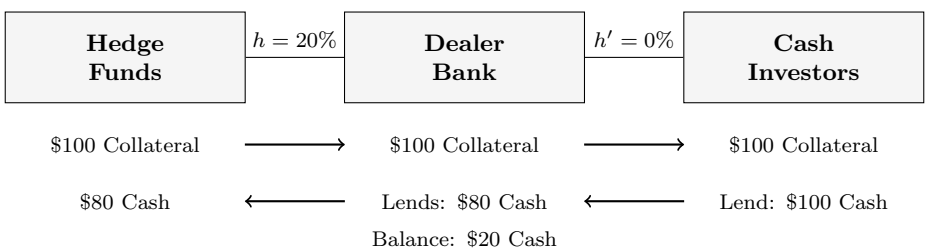

Figure 2: To obtain \$20, the minimum haircut spread needed when cash investors have \$100 is 20%. At the end of the day, cash investors hold the title to the collateral valued at \$100. Hedge funds have \$80 cash. The dealer bank has \$20 cash.

Suppose now that cash investors have \$1000 to lend, while everything else is the same. In that case, \$20 could be obtained by repledging bonds with value \$1000 and charging hedge funds a haircut (spread) of 2%. Figure 3 illustrates that case.

Continuing the example above where cash investors have \$1000 to lend, when the final leg of repo agreement is settled at a later date, the flow of collateral and cash is as in Figure 4.

There are gains from trade for every agent by engaging in repo and reverse repo agreements. The dealer bank serves as an intermediary between cash investors and hedge funds. They earn intermediation profits from interest rate spreads and they can obtain liquidity by haircut differences. As illustrated, this helps the dealer bank to manage liquidity. In period 1, the dealer bank only has its cash holdings as a liquid asset. When short term creditors need to be repaid, shortfall of liquidity could be overcome by rehypothecation and haircut spreads. Cash investors replace short-term creditors. Hence, the dealer can roll

Figure 3: To obtain \$20, the minimum haircut spread needed when cash investors have \$1000 is 2%. At the end of the day, cash investors hold the title to the collateral valued at \$1000. Hedge funds have \$980 cash. The dealer bank has \$20 cash.

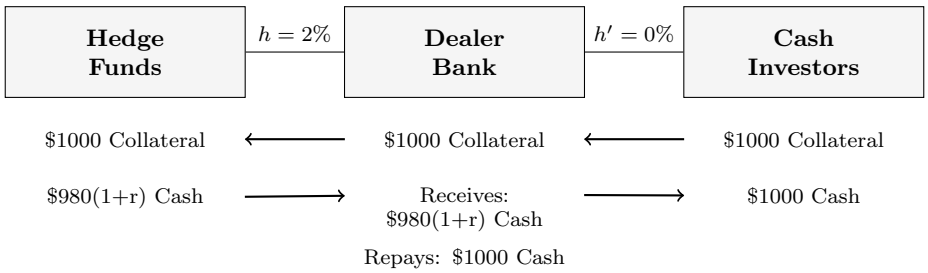

Figure 4: Settlement of the final leg of repo if the dealer bank is not bankrupt. At the end of the day, hedge funds get the title to their collateral valued at \$1000 back. Cash investors are paid \$1000. The dealer bank receives \$980(1+r) from hedge funds and repays \$1000 to cash investors.

over its short-term debt. Cash investors are repaid when the return from the illiquid project is realized. Hedge funds convert bonds into cash and finance their investments. Cash investors store their cash holdings.

3.1 Hedge funds' problem

Hedge funds receive take-it-or-leave-it offers from their dealer bank and other dealer banks, that specify the haircut, the repo interest rate and the amount of bonds that will be repledged, (h_i, r_i, b_i^r). When a hedge fund accepts an offer from a dealer bank, all bonds are given to that dealer bank. The hedge fund receives $b_i^r(1 - h_i)$ dollars and the remaining bonds are kept in custody. When offered contracts with the same expected cash holdings in period 2, they accept the offer of their dealer bank. I assume that any additional bonds that are not repledged are kept in a segregated account with that dealer bank and will be returned to hedge funds, even in the case of bankruptcy. For simplicity, I assume hedge funds are never bankrupt and will always repay their debt, even when the total amount of liability is greater than the value of the collateral.

In period 2, hedge funds get R^H dollars per dollar invested. The dealer

bank is solvent with probability p and the final leg of repo is settled with the dealer bank. Repledged collateral is returned to hedge funds and hedge funds repay what they borrowed plus any interest payments. In addition, hedge funds receive their bonds that were kept in custody.

With probability $1-p$, the dealer bank is bankrupt and the final leg of the repo is not settled with the dealer bank. This gives rise to two cases. First, when the repayment by the hedge fund exceeds the value of collateral backing it. In that case, I assume that when the dealer bank goes bankrupt, a regulator takes over the dealer and hedge funds still need to pay the regulator. Second case arises when the repo liability of the dealer bank exceeds the liability of hedge funds. Hedge funds do not get their repledged collateral back, but they do not repay either. When the dealer bank is bankrupt and the liability of the dealer bank is larger, hedge funds still keep the return from their investment and receive bonds that were not repledged. Hence, they lose the over-collateralized portion of the value of the collateral.

In period 1, each hedge fund i solves the following problem:

$$\max_{\theta_i \in \{0,1\}} \quad \mathbb{1}_{(1-h_i)(1+r_i)\geq 1}\left[\theta_i\left[b_i^r\left((1-h_i)R^H + (1-(1+r_i)(1-h_i))\right) + (b_i - b_i^r)\right] + (1-\theta_i)\,U_H b_i\right]$$
$$+\mathbb{1}_{(1-h_i)(1+r_i)<1}\left[\theta_i\left[b_i^r\left((1-h_i)R^H + p\,(1-(1+r_i)(1-h_i))\right) + (b_i - b_i^r)\right] + (1-\theta_i)\,U_H b_i\right]$$

where $U_H b_i$ is the outside option of hedge funds. If other dealer banks offer better terms for hedge funds' collateral, the outside option is higher. I assume $U_H \geq 1$, because hedge funds can keep all bonds at a custodian without a cost. The dealer bank needs to offer hedge funds at least as much as their outside option, for any transaction to take place between them. Notice the asymmetry in problem. If $(1-h_i)(1+r_i) \geq 1$, then the amount that the hedge fund owes to the dealer bank when the final leg of the repo agreement is settled is greater than the value of the collateral. In that case, the hedge fund is not affected by the bankruptcy of the dealer bank. On the other hand, if $(1-h_i)(1+r_i) < 1$, then the value of the collateral is greater than what the hedge fund owes. The final leg of repo will not be settled and the hedge fund will not recover the difference between the value of the collateral and the amount that the hedge fund owes the dealer bank.

If $(1-h_i)(1+r_i) \geq 1$, then a hedge fund will accept the offer of its dealer bank (That is, $\theta_i = 1$) as long as:

$$b_i^r\left((1-h_i)R^H + (1-(1+r_i)(1-h_i))\right) + (b_i - b_i^r) \geq U_H b_i$$

Rearranging terms, if $(1-h_i)(1+r_i) \geq 1$, a hedge fund accepts the offer as long as:

$$r_i \leq R^H - \frac{\tilde{U}_H}{(1-h_i)} - 1 \tag{1}$$

where $\tilde{U}_H = \frac{b_i(U_H-1)}{b_i^r}$

On the other hand, $(1-h_i)(1+r_i) < 1$, then a hedge fund will accept the offer of its dealer bank (That is, $\theta_i = 1$) as long as:

$$b_i^r \left((1-h_i)R^H + p\left(1-(1+r_i)(1-h_i)\right)\right) + (b_i - b_i^r) \geq U_H b_i$$

Rearranging terms, if $(1-h_i)(1+r_i) < 1$, a hedge fund accepts the offer by the dealer bank as long as:

$$r_i \leq \frac{R^H}{p} - \frac{1-p}{p(1-h_i)} - \frac{\tilde{U}_H}{p(1-h_i)} - 1 \tag{2}$$

where $\tilde{U}_H = \frac{b_i(U_H-1)}{b_i^r}$.

The two curves intersect at $\left(1 - \frac{\tilde{U}_H+1}{R^H}, \frac{R^H}{\tilde{U}_H+1} - 1\right)$ in the (h, r) space. For $h_i < 1 - \frac{\tilde{U}_H+1}{R^H}$, hedge fund participation constraint (1) is relevant and for $h_i > 1 - \frac{\tilde{U}_H+1}{R^H}$, hedge fund participation constraint (2) is relevant as illustrated in Figure 5.

Assumption 1. [11] $\tilde{U}_H + 1 < R^H$

These participation constraints are crucial for the joint determination of haircut spreads and repo interest rate spreads. Net liabilities in the settlement of the final leg of the repo agreement could result in two different regimes. First, if the liability of hedge funds in repo is greater, then for a given haircuts, repo interest rates only depend on hedge fund returns and their outside option. On the other hand, when their net liabilities in repo are smaller, repo interest rates also depend on the probability of bankruptcy of the dealer, since hedge funds are exposed to that risk. For a given haircut spread, dealer banks with a higher probability of being solvent could charge higher repo interest rate spreads and hence receive higher intermediation profits.

In both constraints, the higher the return on the investment projects of hedge funds, R^H, the higher the repo interest rate spreads could be since the dealer bank gets all the surplus. $\tilde{U}_H$ is decreasing in the fraction of bonds to be repledged (b_i^r/b_i). The intuition is that, when more bonds are repledged, hedge funds receive more cash to invest. This partly mitigates losses in case of the

[11] If this assumption does not hold, then the dealer bank has to offer negative interest rates even for infinitesimal haircuts. I make this assumption to make the problem non-trivial.

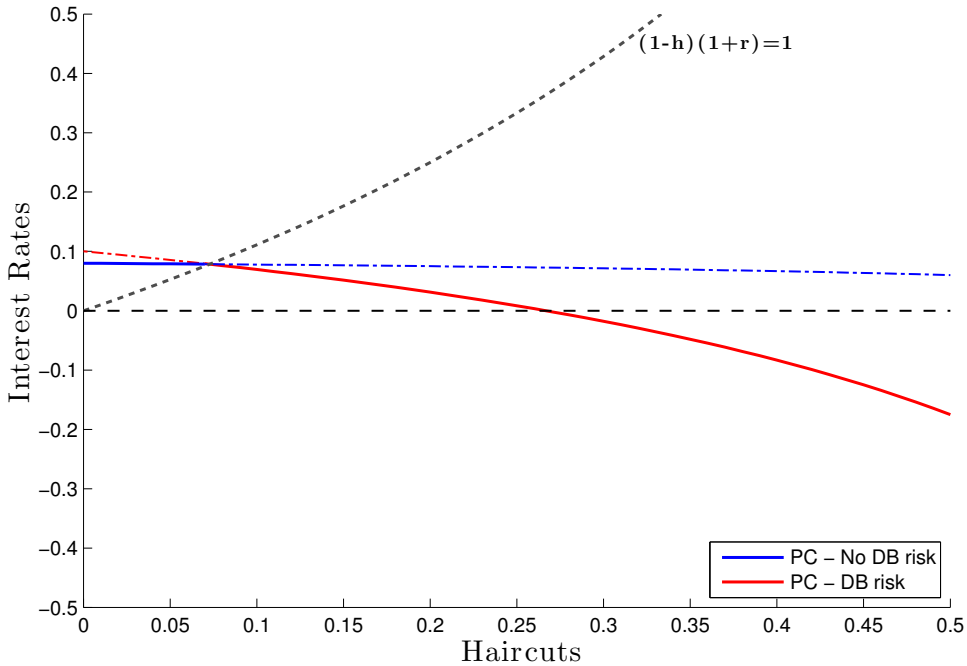

Figure 5: The red curve corresponds to the binding hedge fund participation constraint (2). The blue curve is the binding hedge fund participation constraint (1). The dashed black line has all haircut and interest rate combinations that make the repo liabilities of dealer banks and hedge funds exactly equal to each other. To the left of the dashed black line, net liabilities of hedge funds in repo are larger, hence the participation constraint (1) is relevant. To the right of the dashed black line, net liabilities of the dealer bank in repo are larger, hence the participation constraint (2) is relevant. Parameter values are set to $R^H = 1.1$, $p = 0.8$, $\tilde{U}_H = 0.02$

bankruptcy of the dealer bank, thus a higher interest rate could be sustained. U_H governs the competitiveness of the dealer bank sector. Higher U_H corresponds to a higher outside option for hedge funds, which could be interpreted as more competition in the dealer bank sector. $\tilde{U}_H$ is increasing in U_H. A better outside option for hedge funds lower intermediation profits for the dealer bank. In the limiting case, where $U_H = R^H$, hedge funds can get cash for all their bonds and receive all the surplus.

3.2 The Dealer Bank's problem

The dealer bank faces a liquidity management problem in period 1 to maximize cash holdings in period 2. It makes a take-it-or-leave-it offer to each hedge fund. The offer specifies the haircut, the repo interest rate and the amount of bonds that will be repledged, (h_i, r_i, b_i^r). Furthermore, the dealer bank decides how

much liquidity to store from period 1 to period 2. The proportion of its initial cash holdings to be stored is denoted as α^S. The proportion of liquidity created via rehypothecation and haircut spreads to be stored is denoted by β^S. The dealer bank takes its total assets (E), cash holdings ($(1-\alpha^*)E$), investment in the illiquid project ($\alpha^* E$), and the amount of short-term debt that is maturing (λE) as given. The dealer bank has limited liability, so its payoff is zero in case of bankruptcy. I assume that if the dealer bank's expected cash holdings in period 2 are negative, it automatically goes bankrupt and makes no decisions. The objective function of the dealer bank is:

$$\max_{b_i^r, h_i, r_i, \alpha^S, \beta^S} \quad p\left[R^B \alpha^* E + \alpha^S (1-\alpha^*)E + \beta^S \left(\int_0^1 b_i^r h_i \, di\right) + \int_0^1 \left((1-h_i)(1+r_i) - 1\right) b_i^r \, di\right]$$

The constraints that the dealer bank faces are the following:

The participation constraint of each hedge fund i when $h_i > 1 - (\tilde{U}_H + 1)/R^H$ is represented by (3). Notice that when this constraint binds, there is a negative relationship between haircuts and repo interest rates. Similarly, if the bankruptcy probability of the dealer bank is high, that is p is low, then hedge funds are compensated by lower repo interest rates in order to participate in a repo agreement.

$$(3) \qquad r_i \leq \frac{R^H}{p} - \frac{1-p}{p(1-h_i)} - \frac{\tilde{U}_H}{p(1-h_i)} - 1 \qquad \text{(Hedge Fund Participation)}$$

The participation constraint of each hedge fund i when $h_i \leq 1-(\tilde{U}_H+1)/R^H$ is represented by (4). Notice that when this constraint binds, there is also a negative relationship between haircuts and repo interest rates. In this case, the repayment amount of hedge funds in the final leg is higher than the value of the collateral. Hence, hedge funds are not exposed to the bankruptcy risk of the dealer bank and the participation constraint does not depend on p.

$$(4) \qquad r_i \leq R^H - \frac{\tilde{U}_H}{(1-h_i)} - 1 \qquad \text{(Hedge Fund Participation)}$$

Constraint (5) is the resource constraint. Maturing short term debt to roll over (λE) and the storage of the cash between periods 1 and 2 must be financed by the cash holdings at the beginning of period 1 and liquidity obtained through haircut spreads from cash investors.

(5)

$$\lambda E + \alpha^S \left((1-\alpha^*)E\right) + \beta^S \left(\int_0^1 b_i^r h_i \, di\right) \leq (1-\alpha^*) E + \int_0^1 b_i^r h_i \, di \quad \text{(Resource Constraint)}$$

Constraint (6) is the collateral constraint of the dealer bank with cash investors.[12]

$$\int_0^1 b_i^r \, di \leq Q_m \qquad \text{(Collateral Constraint in Borrowing)} \tag{6}$$

Finally, constraints (7) and (8) are the feasibility constraints.

$$0 \leq \alpha^S \leq 1 \tag{7}$$

$$0 \leq \beta^S \leq 1 \tag{8}$$

In period 1, the dealer bank solves the following problem:

$$\max_{b_i^r, h_i, r_i, \alpha^S, \beta^S} \quad p \left[R^B \alpha^* E + \alpha^S (1-\alpha^*) E + \beta^S \left(\int_0^1 b_i^r h_i \, di \right) + \int_0^1 \left((1-h_i)(1+r_i) - 1 \right) b_i^r \, di \right]$$

subject to (3) for each hedge fund $i \in [0,1]$, (4) for each hedge fund $i \in [0,1]$, (5), (6) and the feasibility constraints (7) and (8).

The objective function of the dealer bank is the total expected cash holdings in period 2. With probability p, the bank will not be bankrupt and its cash holdings will depend on: the total return from the illiquid investment ($R^B \alpha^* E$), total cash that was stored between period 1 and period 2 $\left(\alpha^S (1-\alpha^*) E + \beta^S \left(\int_0^1 b_i^r h_i \, di \right) \right)$ and its net cash position from the settlement of the final leg of repos with hedge funds and cash investors.

Its net cash position from repos is: $\int_0^1 \left((1-h_i)(1+r_i) - 1 \right) b_i^r \, di$. The dealer bank receives $\int_0^1 \left((1-h_i)(1+r_i) \right) b_i^r \, di$ from hedge funds and repays cash investors the amount $\int_0^1 b_i^r \, di$.

That term could be rearranged as: $\int_0^1 \left(r_i (1-h_i) - h_i \right) b_i^r \, di$ where $\int_0^1 \left(r_i (1-h_i) \right) b_i^r \, di$ is profits from intermediation and $\int_0^1 h_i b_i^r \, di$ is the liquidity obtained by haircut spreads in period 1, which needs to be repaid in period 2.

The dealer bank optimally chooses haircut spreads to cover liquidity needs. Higher haircuts could generate liquidity for the dealer bank, but at the expense lower of intermediation profits. In the benchmark model, there are no other tools to obtain liquidity other than the cash holdings and haircut spreads. Therefore, as long as the dealer bank is not bankrupt in period 1 where the expected cash holdings are negative in period 2, it chooses haircut spreads so that the resource constraint is satisfied.

[12] Recall that haircuts are zero in the tri-party repo market between the dealer bank and cash investors.

4 Equilibrium

The equilibrium of this model is defined as the following:

Definition 1. *An equilibrium is a collection of take-it-or-leave-it offers* $\langle h_i^*, r_i^*, b_i^{r*} \rangle$ *made to each hedge fund* $i \in [0,1]$*, participation decisions* θ_i^* *by each hedge fund* $i \in [0,1]$*, and* $\langle \alpha^{S*}, \beta^{S*} \rangle$ *such that:*

- *Hedge funds solve their maximization program in 3.1.*
- *The dealer bank solves its maximization program in 3.2.*

From the viewpoint of the dealer bank, there are multiple offers (h_i, r_i, b_i^r) that yield the same expected cash holdings in period 2. This results in multiplicity of equilibria. For the rest of the paper, I will analyze the symmetric equilibrium where the dealer bank makes the same offer to every hedge fund as assumed in Assumption 2.

Definition 2. *Fraction repledged of the collateral that is permitted to be repledged, denoted by* f_i*, is defined as:* $f_i \equiv b_i^r / b_i$*.*

Assumption 2. *The dealer bank offers contracts to each hedge fund* i *such that for every hedge fund* $i \in [0,1]$*,* $h_i = h$ $r_i = r$ *and* $b_i^r = f b_i$*.*

Assumption 2 makes it the case that, in a symmetric equilibrium, only one haircut, one repo interest rate spread and one fraction of bonds to be repledged need to be pinned down.

Assumption 3. *Hedge funds have weakly more bonds than the amount cash available from cash investors. Namely,* $B \geq Q_m$*.*

Assumption 3 guarantees that all cash coming from cash investors will be used.

Assumption 4. *The dealer bank does not have enough cash holdings to pay the maturing short-term debt. That is,* $\lambda E - (1 - \alpha^*)E \geq 0$

Assumption 4 allows me to focus on the case where haircut spreads will be used as a tool to obtain liquidity in equilibrium.

Lemma 4.1 simplifies the maximization problem of the dealer bank. It asserts that in equilibrium, the dealer bank does not store any liquidity.

Lemma 4.1. *It is not optimal for the dealer bank not to store any liquidity from period 1 to period 2. That is,* $\alpha^{S*} = \beta^{S*} = 0$*.*

Proof. See Appendix. □

Proposition 4.2 allows me to replace the total amount of repledged bonds with the total cash available from cash investors.

Proposition 4.2. *The dealer bank repledges as many bonds as the cash investors lend against. That is,* $\int_0^1 b_i^r \, di = Q_m$.

Proof. See Appendix. □

Before characterizing the dealer bank offers and hedge fund decisions in a symmetric equilibrium, it is useful to define a parameter, which is *the minimum haircut spread needed to cover liquidity needs* by using Assumption 2 and Proposition 4.2.

Definition 3. *The minimum haircut spread needed to cover liquidity needs, denoted as* h_{min}*, is the haircut spread that makes the resource constraint binding when the dealer bank sets* $\alpha^{S*} = \beta^{S*} = 0$*, that is:*

$$h_{min} \equiv \frac{\lambda E - (1 - \alpha^*)E}{Q_m}$$

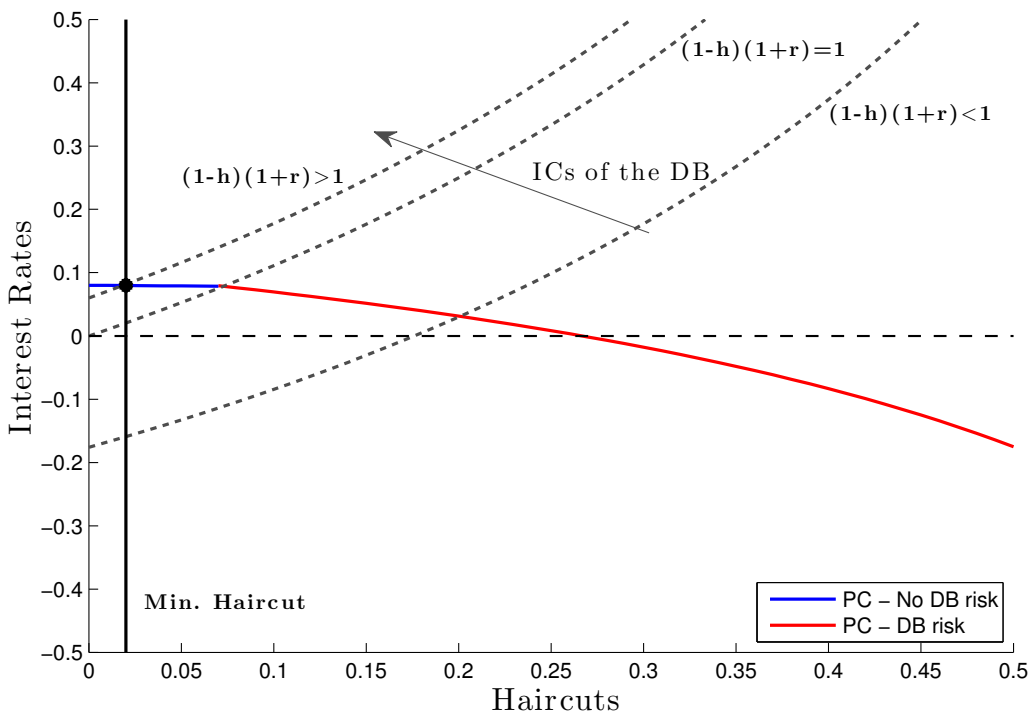

Figure 6: Constraints when $h_{min} \leq 1 - (\tilde{U}_H + 1)/R^H$. Parameter values are set to $R^H = 1.1$, $p = 0.8$, $\tilde{U}_H = (U_H - 1)/f = 0.02$ and $h_{min} = 3\%$

In Figure 6, the vertical line represents the minimum haircut spread needed to cover liquidity needs, the upward sloping curve where $(1 - h)(1 + r) = 1$,

represents the the case when repo liabilities of hedge funds and the dealer bank exactly match. The steep downward sloping curve represents the participation constraint of hedge funds when (3) binds. The flatter downward sloping curve represents the participation constraint of hedge funds when (4) binds. Notice that in equilibrium only one constraint is binding, except when these curves intersect. Solid portions of these lines correspond to when each constraint is binding in equilibrium and the dashed portions of the lines correspond to their slackness.

Figure 6 illustrates the constraints that the dealer bank faces when the minimum haircut spread needed to cover liquidity needs is lower than the haircut spread that is at the intersection of the two hedge fund participation constraints. In this case, the net repo liabilities of hedge funds exceed the net repo liabilities of the dealer bank. I call this case "normal times," when the funding liquidity of the dealer bank is abundant. The proposition below characterizes the offers made by the dealer bank and participation decisions by hedge funds in equilibrium.

Proposition 4.3. *If the funding liquidity of the dealer bank is abundant, such that,*

$$Q_m \geq \frac{R^H(\lambda - (1-\alpha^*))E + (U_H - 1)B}{R^H - 1}$$

then the dealer bank offers every hedge fund contracts (h^*, r^*, b_i^{r*}) *such that:*

- $h^* = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$
- $r^* = R^H - \frac{\tilde{U}_H}{\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - 1$
- $b_i^{r*} = \frac{Q_m}{\int_0^1 b_i \, di} b_i$

and every hedge fund accepts the offer, that is $\theta_i^* = 1$.

Proof. See Appendix. □

Figure 7 illustrates the constraints that the dealer bank faces when the minimum haircut spread needed to cover liquidity needs is higher than the haircut spread that is at the intersection of the two hedge fund participation constraints. In this case, the net repo liabilities of hedge funds are lower than the net repo liabilities of the dealer bank. I call this case "distressed times," when the funding liquidity of the dealer bank is relatively scarce. The proposition below characterizes the offers made by the dealer bank and participation decisions by hedge funds in equilibrium.

Proposition 4.4. *If the funding liquidity of the dealer bank is relatively scarce, such that,*

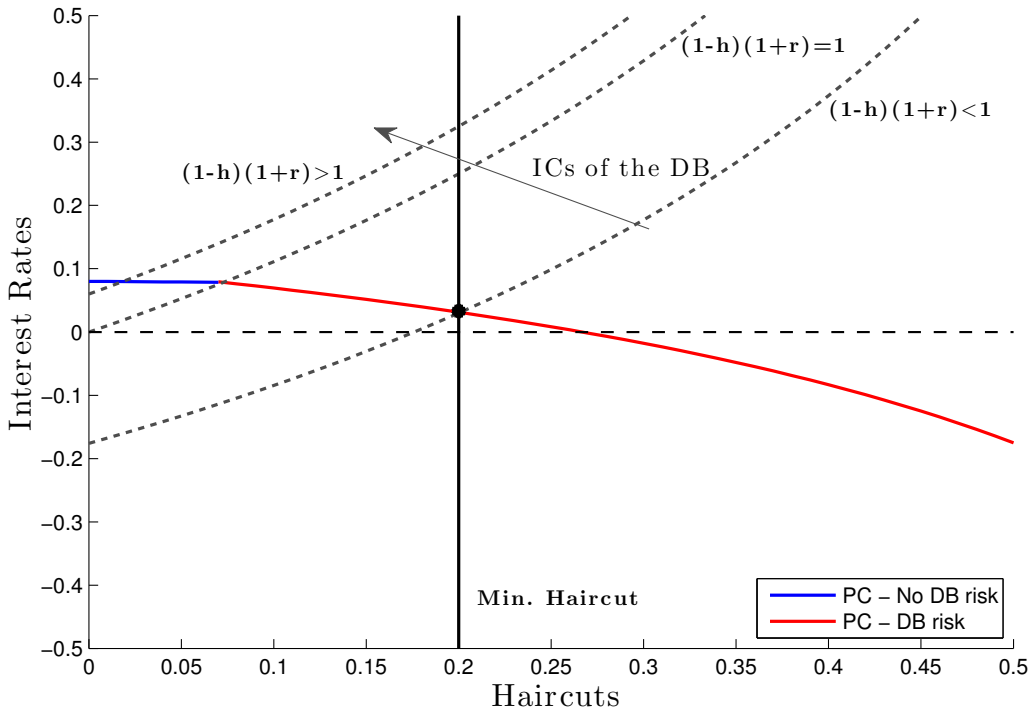

Figure 7: Constraints when $h_{min} > 1 - (\tilde{U}_H + 1)/R^H$. Parameter values are set to $R^H = 1.07$, $p = 0.8$, $\tilde{U}_H = 1$ and $h_{min} = 20\%$

$$Q_m \in \left[\frac{R^H \left(\lambda - (1-\alpha^*) - pR^B\alpha^*\right) E + (U_H - 1)B}{R^H - 1}, \frac{R^H(\lambda - (1-\alpha^*))E + (U_H - 1)B}{R^H - 1} \right)$$

then[13]*:*

- $h^* = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$
- $r^* = \frac{R^H}{p} - \frac{1-p}{p\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - \frac{\tilde{U}_H}{p\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - 1$
- $b_i^{r*} = \frac{Q_m}{\int_0^1 b_i \, di} b_i$

and every hedge fund accepts the offer, that is $\theta_i^* = 1$.

Proof. See Appendix. □

[13] I assume that when the dealer bank has expected cash holdings of zero, it is not automatically bankrupt

Corollary 1. *In the symmetric equilibrium, the fraction of the collateral that is repledged is increasing in the cash available from cash investors, that is* $df/dQ_m > 0$.

Proof. In the symmetric equilibrium, $b_i^{r*} = \frac{Q_m}{\int_0^1 b_i \, di} b_i$. Hence, $df/dQ_m = \frac{1}{\int_0^1 b_i \, di} > 0$.

□

Finally, Proposition 4.5 gives the condition that the dealer bank cannot profitably manage liquidity in period 1 and goes bankrupt.

Proposition 4.5. *If the funding liquidity of the dealer bank is extremely scarce, such that:*

$$Q_m < \frac{R^H \left(\lambda - (1-\alpha^*) - pR^B\alpha^*\right) E + (U_H - 1)B}{R^H - 1}$$

then the dealer bank cannot manage liquidity while having non-negative expected cash holdings in period 2. Hence, the dealer goes bankrupt in period 1 before making any decisions.

Proof. See Appendix. □

5 Discussion of the Benchmark Model

In the benchmark model, hedge funds have bonds to use as collateral to finance their investments. The dealer bank serves as an intermediary between hedge funds and cash investors. It repledges hedge fund collateral to cash investors. Given its favorable position as an intermediary, it earns intermediation profits by the difference between interest rates in the markets that they participate. Moreover, it can obtain liquidity to use for its own purposes by haircut spreads. Two different regimes that are shown in Proposition 4.3 and Proposition 4.4 provide interesting novel intuitions and comparative statics about haircuts and repo rates.

Proposition 4.3 characterizes the "normal times," when funding liquidity of a dealer bank is abundant. In normal times, hedge funds are not exposed to the bankruptcy risk of the dealer bank. Funding liquidity of the dealer bank is abundant when the total amount of maturing debt is low, the cash holdings are high or the amount of funds available from cash investors is high. In normal times, the dealer bank does not need to higher haircut spreads aggresively to improve its liquidity position. Since cash available from cash investors is high, the dealer bank is able to cover its liquidity needs by lower haircut spreads. Furthermore, since lower haircuts correspond to higher repo interest rates, the dealer bank is able to generate higher intermediation profits via interest rate spreads. The model predicts that in normal times, the repo liability of hedge funds exceed the repo liability of the dealer bank. Hence, hedge funds do not face

the bankruptcy risk of the dealer. Through the rehypothecation chain, funding liquidity of the dealer bank, that depends on the maturing short-term debt, the cash holdings and the cash availability from cash investors will propagate to hedge funds by means of low haircuts and larger degree of availability of leverage to the hedge fund sector.

On the other hand, Proposition 4.4 corresponds to "distressed times" when funding liquidity of the dealer bank is relatively scarce and hedge funds are exposed to the bankruptcy risk of the dealer bank. Thus, the exogenous probability of bankruptcy is relevant in determining the repo interest rates. In distressed times, larger short-term debt that is maturing, lower cash holdings or less availability of cash from cash investors correspond to higher haircuts charged to hedge funds. Since higher haircuts expose hedge funds to more risk in the event of default by the dealer bank, hedge funds are compensated through lower repo rates. This leads to lower intermediation profits.

Figure 8 illustrates the binding participation constraints of hedge funds for different exogenous probabilities of bankruptcy. In normal times, the bankruptcy probability of the dealer does not affect interest rates. Higher haircuts only increase the attractiveness of the outside option of the hedge funds. However, in distressed times, hedge funds are exposed to the bankruptcy risk of the dealer bank. The larger the bankruptcy probability, the more hedge funds need to be compensated through a reduction in the repo interest rates.

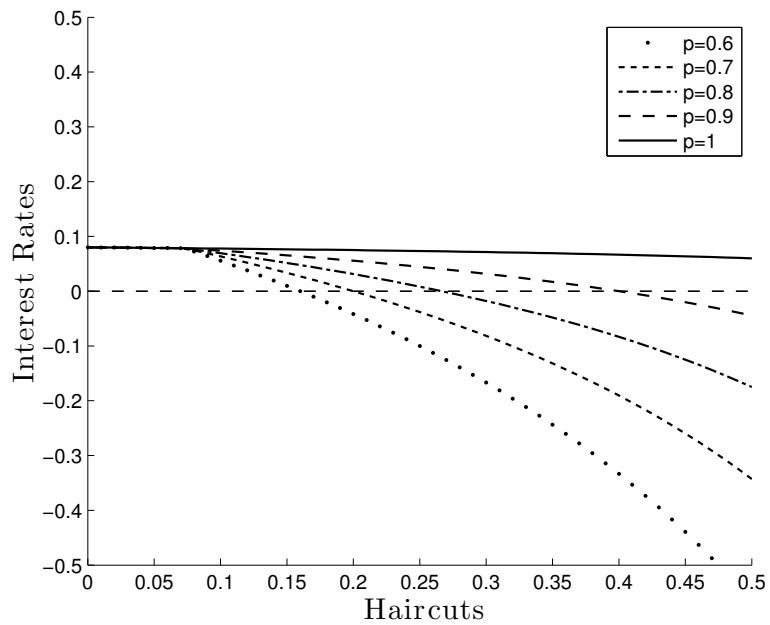

Figure 8: Comparative Statics of the Exogenous Probability of Bankruptcy. Parameter values are set to $R^H = 1.1$ and $\tilde{U}_H = 0.02$

Corollary 1 follows naturally since the total collateral demanded from the hedge funds is equal to the total cash available to the dealer bank from cash investors. Since the dealer bank has no access to other markets to use the additional collateral, it is segregated and kept in custody. Hence, fraction repledged of the collateral that is permitted to be repledged is increasing in the cash available from cash investors.

In the model, negative intermediation profits could be optimal. If the return from illiquid assets is high enough, it might be optimal to offer a negative interest rate. In that case, the dealer bank is effectively borrowing from hedge funds. However, if the minimum haircut spreads needed to cover liquidity needs is above a certain threshold, then the dealer bank will not be able to satisfy the resource constraint while remaining profitable. In that case, the bank will be forced to go bankrupt.

6 Extensions of the Benchmark Model

6.1 Allowing the Dealer Bank to Liquidate the Illiquid Project

In this section, I extend the benchmark model to allow for fire sales of illiquid assets of the dealer bank. When I allow for fire sales, the resource constraint of the dealer bank becomes:

(9)

$$\lambda E + \alpha^S \left((1-\alpha^* + \alpha^L \alpha^* l)E\right) + \beta^S \left(\int_0^1 b_i^r h_i \, di\right) \leq \left(1 - \alpha^* + \alpha^L \alpha^* l\right) E + \int_0^1 b_i^r h_i \, di \quad \text{(RC)}$$

When the dealer bank liquidates the illiquid asset, the right hand side becomes higher compared to the case with no liquidation. Sales of assets generate extra liquidity of $l < 1$ per unit sold. The dealer bank liquidates illiquid assets valued at αE. Hence, the total liquidity obtained by liquidation is $l\alpha^* E$.

When fire sales of illiquid assets is allowed, the dealer bank solves the following problem in period 1:

$$\max_{b_i^r, h_i, r_i, \alpha^L, \alpha^S, \beta^S} \quad p\left[R^B(1-\alpha^L)\alpha^* E + \alpha^S(1-\alpha^* + \alpha^L \alpha^* l)E + \int_0^1 \left(r_i(1-h_i) - h_i\right) b_i^r \, di\right]$$

subject to (3) for each hedge fund $i \in [0,1]$, (4) for each hedge fund $i \in [0,1]$, (9), (6), the feasibility constraints (7), (8) and $\alpha^L \in \{0,1\}$.

In this extension, fire sales is an additional source to obtain liquidity. I assume that the fire sales value of the illiquid asset is $l < 1$ per unit sold. All other parts of the problem are the same as in the benchmark model, except the objective function, the resource constraint and the additional feasibility constraint

on α^L. In this problem, the ways to cover liquidity needs are the cash holdings, haircut spreads and fire sales. In reality, when liquidity is abundant, fire sales are not necessary. In this extension, I will focus on the case when liquidity is relatively scarce and the dealer bank decides between several alternatives to manage liquidity. After using all the cash holdings necessary to cover liquidity needs, fire sales reduce the need to create liquidity via haircut spreads. Hence, the dealer bank could obtain higher intermediation profits. However, this comes at the expense of foregone returns from the illiquid project. In equilibrium, the dealer bank weighs the costs and benefits of fire sales and decides optimally the extent to use each funding mechanism.

In this extension, equilibrium is defined as:

Definition 4. *An equilibrium is a collection of take-it-or-leave-it offers* $\langle h_i^*, r_i^*, b_i^{r*}\rangle$ *made to each hedge fund* $i \in [0,1]$*, participation decisions* θ_i^* *by each hedge fund* $i \in [0,1]$*, and* $\langle \alpha^{L*}\alpha^{S*}, \beta^{S*}\rangle$ *such that:*

- *Hedge funds solve their maximization program.*
- *The dealer bank solves its maximization program when fire sales is possible.*

Assumption 5. *Cash available from cash investors is low enough, so that after fire sales the net repo liabilities of the dealer is higher and constraint (3) binds. That is,* $Q_m > \frac{R^H(\lambda-(1-\alpha^*+\alpha^* l))E+(U_H-1)}{R^H-1}$

In this extension, I keep the Assumptions 2, 3 and replace Assumption 4 with Assumption 5.

Assumption 5 allows me to characterize equilibrium when only the participation constraint when the repo liabilities of the dealer bank exceeds the repo liabilities of hedge funds. The assumption guarantees that the interest rate regime does not change. To solve for other cases, this assumption could be relaxed.

Equilibrium offers and participation decisions when liquidity is scarce, but fire sales value of illiquid assets is relatively high is characterized in Proposition 6.1.

Proposition 6.1. *If Assumption 5 holds and* $l > p\frac{R^B}{R^H}$*, then the dealer bank offers every hedge fund:*

- $h^* = \frac{\lambda E-(1-\alpha^*+\alpha^* l)E}{Q_m}$
- $r^* = \frac{R^H}{p} - \frac{1-p}{p\left(1-\frac{\lambda E-(1-\alpha^*+\alpha^* l)E}{Q_m}\right)} - \frac{\tilde{U}_H}{p\left(1-\frac{\lambda E-(1-\alpha^*+\alpha^* l)E}{Q_m}\right)} - 1$
- $b_i^{d*} = \frac{Q_m}{\int_0^1 b_i\, di} b_i$

and every hedge fund accepts, that it $\theta_i = 1$*.*

Furthermore, the dealer bank liquidates its illiquid assets, hence $\alpha^{L*} = 1$*.*

Proof. See Appendix. □

Equilibrium offers and participation decisions when liquidity is scarce and the fire sales value of illiquid assets is relatively low is characterized in Proposition 6.1.

Proposition 6.2. *If Assumption 5 holds and* $l \leq p\frac{R^B}{R^H}$*, then the dealer bank offers every hedge fund:*

- $h^* = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$
- $r^* = \frac{R^H}{p} - \frac{1-p}{p\left(1-\frac{\lambda E-(1-\alpha^*)E}{Q_m}\right)} - \frac{\tilde{U}_H}{p\left(1-\frac{\lambda E-(1-\alpha^*)E}{Q_m}\right)} - 1$
- $b_i^{d*} = \frac{Q_m}{\int_0^1 b_i\, di} b_i$

and every hedge fund accepts, that it $\theta_i = 1$*.*

Furthermore, the dealer bank does not liquidate its illiquid assets, hence $\alpha^{L*} = 0$*.*

Proof. See Appendix. □

When liquidity is scarce and the opportunity cost of fire sales is relatively low, then the dealer bank liquidates the illiquid assets, charges a lower haircut spread and captures higher intermediation profits. On the other hand, when liquidity is scarce and the opportunity cost of liquidation is high, then the dealer bank sets higher haircut spreads and hold on to its illiquid assets. If foregone investment returns are higher than foregone intermediation profits without liquidation, fire sales and higher haircut spreads would be optimal.

7 Discussion of the Model and Empirical Evidence

In this section, I compare the predictions of the model to the descriptive evidence on haircut spreads, the size of the tri-party market and dealer bank balance sheets. Figure 9 shows the size of the tri-party market. In line with the predictions of the model, net repo financing of dealer banks increase during the boom and decrease during the crisis and onwards, essentially tracking the amount of funding available in the tri-party market as shown in Figure 10. One central prediction of the model is that, the dealer banks are able to increase their net repo financing when funding liquidity is abundant with low haircut spreads. On the other hand, when funding liquidity is scarce, all else constant, haircut spreads must increase. Comparative statics of the model suggest that as liquidity needs of the dealer bank declines, haircut spreads would decline. This would happen if the dealer banks de-lever, increase their liquidity holdings or the liquidation value of assets increase.

Table 1 documents the variation of haircuts that different counter-parties face in June 2007 and June 2009 obtained from CGFS [2010]. Haircuts that

Figure 9: *Source: Fahy and Martin [2012], Tri-Party Repo Infrastructure Reform Task Force, The FSOC Report (2011).* Limited data were provided by clearing banks prior to April 2008. Estimates are based on the data provided.

dealer banks faced in June 2007 and June 2009 when they borrowed is documented under the prime category.[14] On the other hand, haircuts that hedge funds faced in June 2007 and June 2009 are documented under the unrated category. Notice that the haircut spreads across all securities increased conditional on the securities being accepted as collateral both in June 2007 and in June 2009, in line with the predictions of the model.[15]

Figure 11 shows the extent of rehypothecation used by each dealer bank obtained from 10Q and 10K filings of the dealer banks. It illustrates the rapid increase in the volume of collateral that the dealer banks repledged between 2001 and 2008 and a sharp decline during the crisis. Furthermore, between 2009 and 2014, the volume of repledged collateral remained stable and lower than the pre-crisis levels for Goldman Sachs, Morgan Stanley and Bank of America/Merrill Lynch. The model suggests that the reduction of the availability of cash from cash investors reduces the possibility of repledging collateral, hence reducing the volume of repledged collateral. The evidence presented here is line with the

[14] Primary dealers serve as trading counterparties of the New York Fed in its implementation of monetary policy.

[15] A haircut of 100% corresponds to a security not being accepted as collateral.

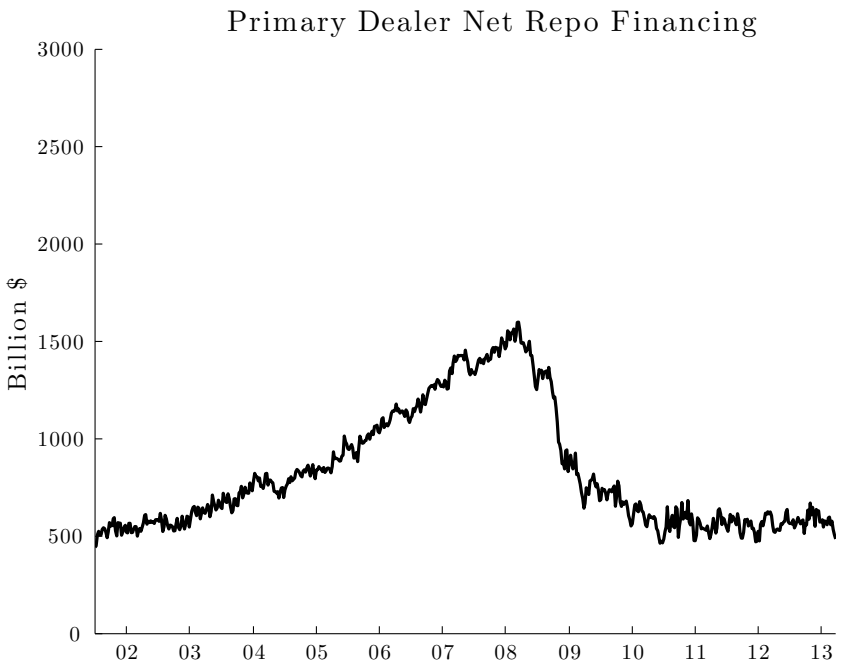

Figure 10: *Source: FR 2004C*. Primary Dealers' Net Repo Financing. The figure shows the value of the repos minus reverse repos of the primary dealers.

predictions of the model.

Another important feature of the data is that, in the second quarter of 2008, the fair value of collateral that Lehman Brothers was permitted to repledge and the fair value of collateral that it did repledge were around a half their values in the first quarter and the second quarter of 2008. This could be attibuted to a run on collateral as argued by Duffie [2010]. Furthermore, the fraction that is repledged reduced from 92% in the first quarter of 2008 to 82% in the second quarter of 2008. The reduction in the total value of collateral and the fraction repledged could also be explained by the model, since the total lending by cash investors to Lehman Brothers reduced during that time.

To reconcile the evidence on increasing haircut spreads and decreasing volume of rehypothecation with the model, I consider the following example. In the third quarter of 2008, the fair value of the collateral that Goldman Sachs received was $831 billion and Goldman Sachs repledged $691 billion of that collateral. Similarly, in the third quarter of 2008, Morgan Stanley received collateral at fair value $953 billion and repledged $711 billion. To obtain, say $50 billion in cash, Goldman Sachs needed a haircut spread of 7.2% and Morgan Stanley needed a haircut spread of 7%.[16]

[16]There is no data available on the composition of the pool of repledged collateral. This

Table 1: Typical haircuts on term securities financing transactions (per cent)

	June 2007			June 2009		
	Prime	Non-prime	Unrated	Prime	Non-prime	unrated
G7 government bonds						
Short-term	0	0	0.5	0.5	1	2
Medium-term	0	0	0.5	1	2	3
US agencies						
Short-term	1	2	3	1	2	3
Medium-term	1	2	3	2	5	7
Pfandbrief	0	0	1	1	2	8
Prime MBS						
AAA-rated	4	6	10	10	20	30-100
AA and A-rated	8	12	25	100	100	100
ABS	10	20	20	25	50	100
Structured (AAA)	10	15	20	100	100	100
Investment grade bonds						
AAA & AA- rated	1	2	5	8	12	15
A & BBB-rated	4	7	10	10	15	20
High-yield bonds	8	12	20	15	20	40
Equity						
G7	10	12	20	15	20	25
Emerging	15	20	35	20	25	40

Notes: Typical haircuts that prime counterparties, non-prime counterparties and hedge funds and other unrated counterparties faced in June 2007 and June 2009. A haircut of 100% means that the collateral is not accepted.
Source: CGFS [2010].

In the fourth quarter of 2008, Goldman Sachs received collateral with a fair value of $578 billion, of which it repledged $445 billion, whereas Morgan Stanley received collateral with a fair value of $294 billion, of which it repledged $227 billion. All else equal, that corresponds a need of a haircut spread of 11.2% for Goldman Sachs and a haircut spread of %22 for Goldman Sachs to obtain $50 billion. The model predicts a sharper reduction in assets and a sharper reduction in lending to hedge funds for Morgan Stanley than for Goldman Sachs.

Figure 12 illustrates the total assets of the dealer banks between the first quarter of 2001 and the first quarter of 2014. The predictions of the model are supported by the data. The total assets of Goldman Sachs reduced from $1081 billion to $884 billion, whereas the total assets of Morgan Stanley reduced from $987 billion to $658 billion between the third and fourth quarters of 2008. This evidence is in line with the story that Morgan Stanley needed a higher haircut spread to obtain liquidity. However, charging a high haircut spread

example suggests that the haircut spread needed is a weighted average, where the weights are determined by the composition of the collateral pool.

while keeping their prime brokerage clients proved to be difficult and they had to de-lever by the fire sales of assets.

Figure 13 shows the receivables of the dealer banks from their customers. Margin loans given to their clients are recorded as receivables in the balance sheets of the dealers.[17] Dealer banks lend to hedge funds mostly in collateralized agreements such as repos and margin loans. Hedge funds fund their long positions by getting a margin loan from their prime brokers and pledge securities bought as collateral. Even though, margin loans and repos have legally distinct from each other, margin loans are also a form of collateralized lending.[18]

Figure 13 suggests that greater availability of funding liquidity to the dealer banks in a boom and the scarcity of funding liquidity to the dealer banks in a crisis is reflected on hedge funds. Continuing the discussion above on Goldman Sachs and Morgan Stanley, the data suggests that the amount of funding liquidity available to dealer banks has an impact on lending to hedge funds. At its peak in the fourth quarter of 2007, Goldman Sachs had receivables worth $129 billion and receivables declined to $64 billion in the fourth quarter of 2008. On the other hand, Morgan Stanley had receivables worth $113 billion at its peak in the third quarter of 2007, which declined to $31 billion in the fourth quarter of 2008. Hence, the model can provide an explanation for the evidence documented in Aragon and Strahan [2012] and Klaus and Rzepkowski [2009].

[17] Collateral obtained from margin loans is not recorded in the balance sheets.

[18] One legal distinction, for example, is that, under the SEC rule 15c3-3 in the US, for each $100 cash lent by margin loans, the dealer is permitted to rehypothecate up to $140 worth of the client's assets. There is no such restriction for repo agreements. In the UK, there is no limit on the rehypothecation of client assets. For a more detailed discussion, see Duffie [2010].

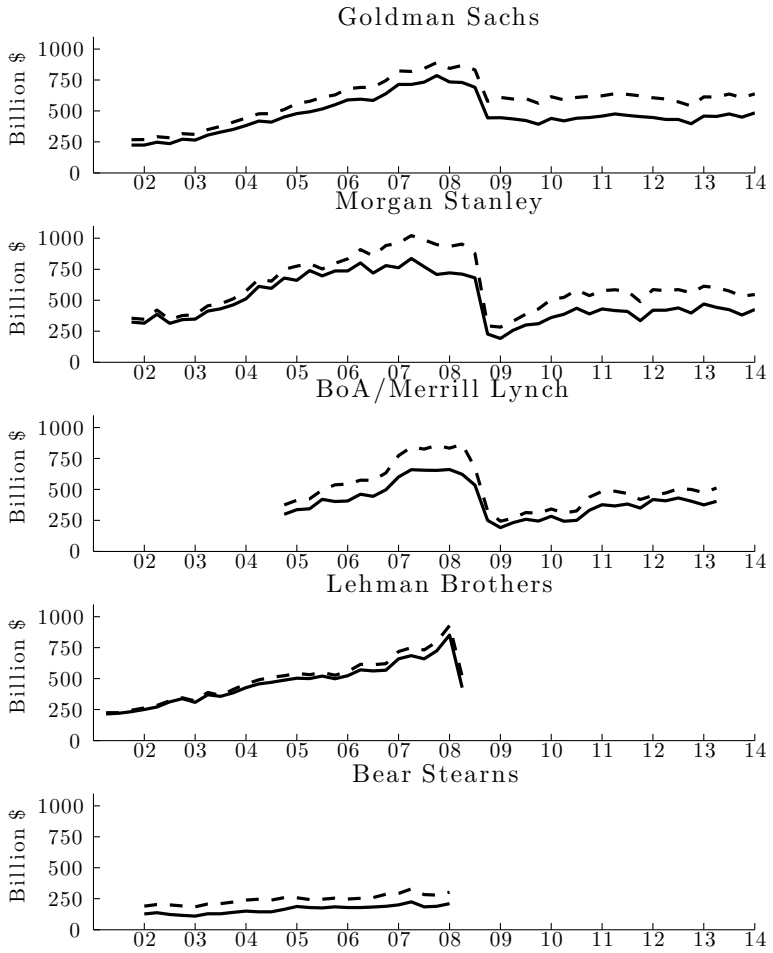

Figure 11: The total value of collateral that dealer banks were permitted to repledge (Dashed Lines) and the total value of collateral that dealer banks repledged. (Solid Lines) The data is obtained from the 10Q and 10K filings of the dealer banks.

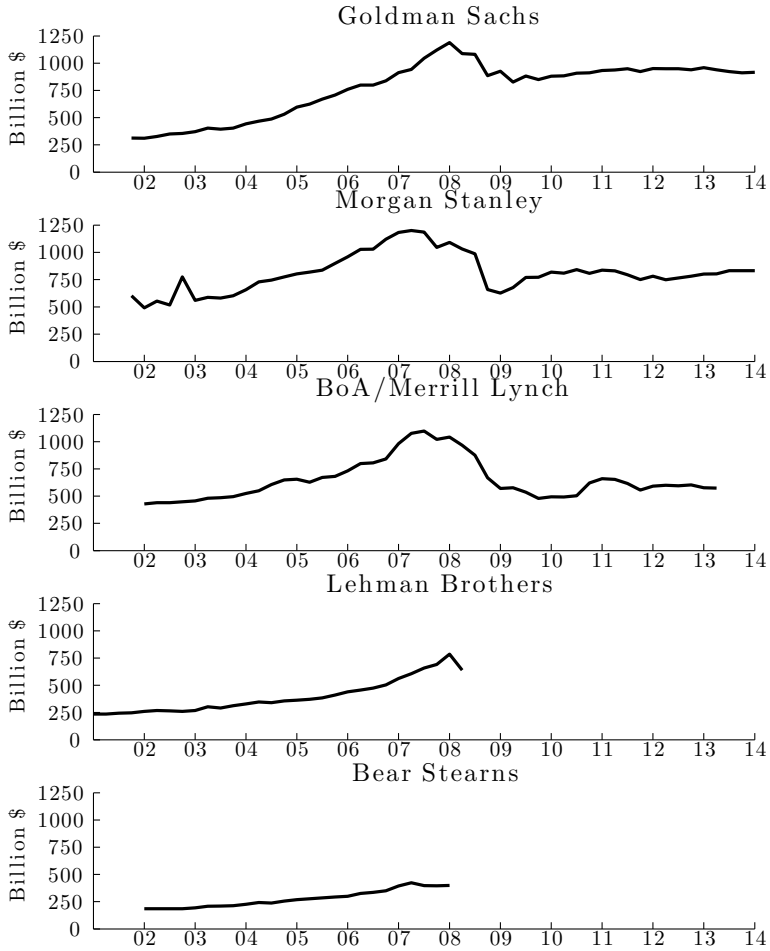

Figure 12: Total assets. *Source: 10Q and 10K filings.*

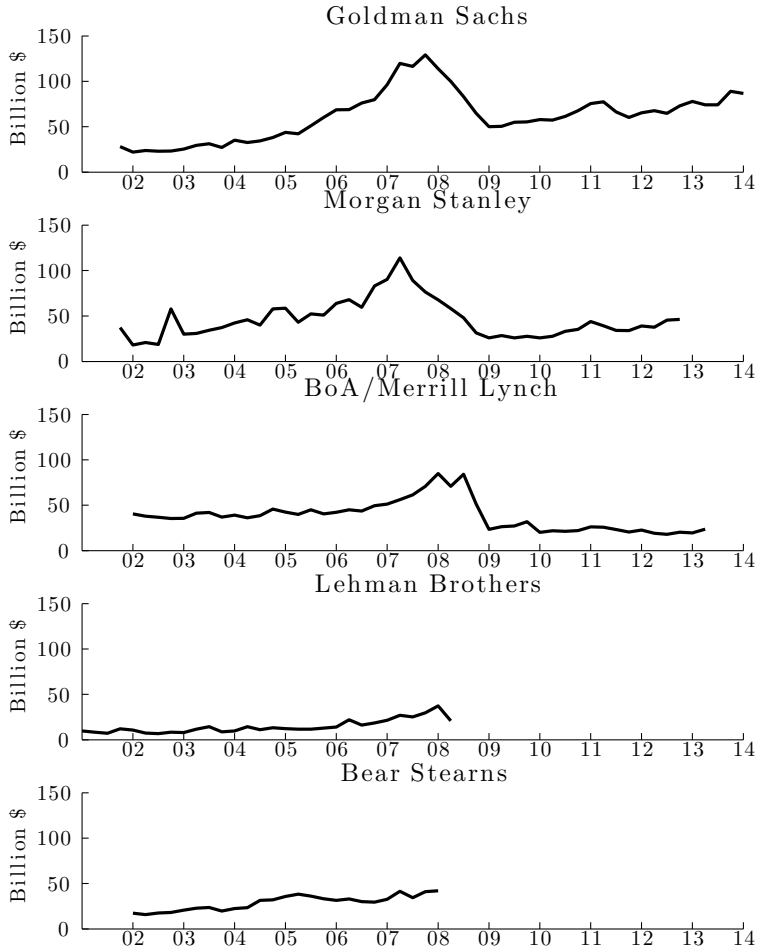

Figure 13: Total Receivables from Customers. *Source: 10Q and 10K filings.*

8 Conclusion

In this paper, I proposed a new mechanism that determines haircuts and interest rates in repo markets by dealer bank funding liquidity and rehypothecation. I argued that dealer banks can intermediate the provision of funding to hedge funds by repledging hedge fund collateral to cash investors. Furthermore, by charging higher haircuts to hedge funds than those of their repos with cash investors, dealer banks obtain extra funding for themselves, to the extent of the difference in haircuts. I showed how and when dealer banks use this funding mechanism over alternatives such as cash holdings and fire sales of illiquid assets. The main message of this paper is the following. The total amount of liquidity provided by cash investors, dealer banks' balance sheets, competitiveness of the dealer bank sector and hedge fund characteristics determine haircuts and repo interest rates. In normal times, haircut spreads are low and hedge funds are not exposed to the bankruptcy risk of dealer banks. In distressed times, when liquidity provided by cash investors is relatively scarce, then haircut spreads are high. Furthermore, lending to hedge funds by dealer banks is over-collateralized. Rehypothecation and over-collateralization of loans expose hedge funds to the bankruptcy risk of dealer banks. Therefore, hedge funds must be compensated by lower repo rates in equilibrium. Furthermore, in the model, sudden liquidity dry-up of funding liquidity of dealer banks can easily trigger bankruptcy. Dry-up of dealer bank funding liquidity propagates to hedge funds by means of higher haircuts. An important feature of normal times is that dealer banks and hedge funds have high leverage. On the other hand, distressed times, when funding liquidity of dealer banks is relatively scarce, are characterized by low leverage for hedge funds.

This paper suggests that cash investors are the key in determining haircuts and leverage in financial markets. Money market mutual funds, dealer banks and hedge funds were at the center of the financial crisis. Six years after the financial crisis, the risks involved in the lending-borrowing relationships of these institutions remain mostly intact. The financial system is still vulnerable to liquidity dry-ups and wild increases in haircuts. This paper highlights the need for further theoretical and empirical research that will be of interest to both academics and policy makers to develop tools to mitigate risks in the financial system.

A Appendix

Proof. (***Lemma 4.1)***
Using Assumption 2, the dealer bank maximizes cash holdings in period 2. Note that I drop p for notational simplicity as it does not change the results.

$$R^B\alpha^* E + \alpha^S(1-\alpha^*)E + \beta^S hfB + ((1+r)(1-h)-1)\,fB$$

where $f = b_i^r/b_i$ and $B = \int_0^1 b_i\,di$.

The constraints in the problem of the dealer bank are:

$$R^H - \frac{\tilde{U}_H}{(1-h)} - 1 - r \geq 0 \qquad (\mu_1) \tag{10}$$

$$\frac{R^H}{p} - \frac{1-p}{p(1-h)} - \frac{\tilde{U}_H}{p(1-h)} - 1 - r \geq 0 \qquad (\mu_2) \tag{11}$$

$$\lambda E + \alpha^S\left((1-\alpha^*)E\right) + \beta^S hfB \leq (1-\alpha^*)\,E + hfB \qquad (\mu_3) \tag{12}$$

$$Q_m - fB \geq 0 \qquad (\mu_4) \tag{13}$$

$$0 \leq \alpha^S \leq 1 \qquad (\mu_5, \mu_6) \tag{14}$$

$$0 \leq \beta^S \leq 1 \qquad (\mu_7, \mu_8) \tag{15}$$

where $\mu_1, \mu_2, ..., \mu_{10}$ are the respective Lagrange multipliers.
The first order conditions with respect to α^S and β^S are:

$$(1-\alpha^*)E(1-\mu_3) + \mu_5 - \mu_6 = 0$$
$$hfB(1-\mu_3) + \mu_7 - \mu_8 = 0$$

respectively.
The first order condition with respect to h, when (10) is binding yields:

$$\mu_3 = \frac{R^H - \beta^S}{1-\beta^S}$$

In that case, suppose $\beta^S = 1$. μ_3 is increasing in β^S. As β^S goes to 1, μ_3 tends to infinity. Furthermore, when $\beta^S = 1$, the resource constraint is not satisfied. Reducing β^S would relax the resource constraint and result in higher cash holdings as measured by μ_3. $\beta^S = 0$ would make μ_3 obtain its lowest possible value, but still greater than 1. Therefore, $\mu_7 > 0$ and $\beta^{S*} = 0$. From its first order condition: $\mu_7 = hfB(R^H - 1)$ and $\mu_8 = 0$.

Similarly, plugging in the value of μ_3 in the first order condition for α^S: $\mu_5 = (1-\alpha^*)E(R^H - 1)$ and $\mu_6 = 0$ meaning that $\alpha^{S*} = 0$.

A similar argument holds when (11) is binding instead of (10).

□

Proof. (***Proposition 4.2)***
In equilibrium, $\mu_3 > 0$ as shown in Lemma 4.1. It means that the resource

constraint is binding. Furthermore, using the fact that there is no storage between period 1 and period 2, namely $\alpha^{S*} = \beta^{S*} = 0$, in equilibrium, $h^* = \kappa/fB$, where $\kappa \equiv \lambda E - (1-\alpha^*)E$. Since, between (11) and (10) only one binds (except for when they intersect), I will show that $\int_0^1 b_i^r \, di = Q_m$ for both cases. Note that $\int_0^1 b_i^r \, di = fB$

First, if h is such that (10) binds, by replacing h^* and $r*$ in the objective function, the only constraint in the problem becomes (13).

The objective function is the following:

$$R^B \alpha^* E + ((1+r)(1-h) - 1)\, fB$$

Note that when (10) is binding:

$$(1+r)(1-h) = \left(R^H - \frac{U_H - 1}{f(1-h)}\right)(1-h)$$

Note that I replaced $\tilde{U}_H$ by $(U_H - 1)/f$ which are identical. By distributing $(1-h)$ and plugging in $h = \kappa/fB$:

$$(1+r)(1-h) = R^H \left(1 - \frac{\kappa}{fB}\right) - \frac{U_H - 1}{f}$$

Then the problem is to maximize:

$$R^B \alpha^* E + \left(R^H \left(1 - \frac{\kappa}{fB}\right) - \frac{U_H - 1}{f} - 1\right) fB$$

subject to (13).

The first order condition with respect to f is:

$$(R^H - 1)B - \mu_4 = 0$$

Hence, in equilibrium $\mu_4 = (R^H - 1)B > 0$ and (13) is binding.

Following similar steps, in the case when (11) is binding, $\mu_4 = (R^H/p - 1)B > 0$.

Therefore, $\int_0^1 b_i^r \, di = Q_m$. □

Proof. (***Proposition 4.3)***
Using the result in Proposition 4.3, I replace f with Q_m/B. This proposition characterizes the offers and participation decisions when the net repo liabilities of hedge funds are higher. That happens when $h_{min} \leq 1 - \frac{\tilde{U}_H + 1}{R^H}$. The condition:

$$Q_m \geq \frac{R^H(\lambda - (1-\alpha^*))E + (U_H - 1)B}{R^H - 1} \tag{16}$$

is equivalent to that but it is rearranged.

If the condition (16) holds, then funding liquidity of the dealer bank is abundant and hedge funds' repo liability exceed the liability of the dealer bank. Since the objective function of the dealer bank is decreasing in haircuts, haircuts offered are equal to $h^* = h_{min} = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$.

Plugging in the optimal haircut into participation constraint (4), equilibrium interest rates are:

$$r^* = R^H - \frac{\tilde{U}_H}{\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - 1$$

Finally, $b_i^{r*} = \frac{Q_m}{\int_0^1 b_i \, di} b_i$ follows from Proposition 4.2.

When offered this contract, hedge funds are indifferent. Hence, they choose to participate. Therefore, $\theta_i = 1$. □

Proof. (***Proposition 4.4)***
This proposition characterizes offers and participation decisions when liquidity is relatively scarce and the repo liabilities of the dealer bank exceed the repo liabilities of hedge funds.

That happens when $h_{min} > 1 - \frac{\tilde{U}_H + 1}{R^H}$. This provides the upper bound on the condition:

$$Q_m \in \left[\frac{R^H \left(\lambda - (1-\alpha^*) - pR^B\alpha^*\right) E + (U_H - 1)B}{R^H - 1}, \frac{R^H(\lambda - (1-\alpha^*))E + (U_H - 1)B}{R^H - 1} \right)$$

The lower bound is when the dealer bank makes zero expected profits in which case, it goes bankrupt without making offers. That lower bound is found in Proposition 4.5.

If this condition on Q_m holds, in equilibrium, hedge fund participation constraint (3) binds. The objective function is decreasing in h, hence h_{min} will be offered in equilibrium.

Therefore,

- $h^* = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$
- $r^* = \frac{R^H}{p} - \frac{1-p}{p\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - \frac{\tilde{U}_H}{p\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right)} - 1$
- $b_i^{r*} = \frac{Q_m}{\int_0^1 b_i \, di} b_i$

where $h^* = h_{min}$, r^* is found from hedge fund participation constraint (3) and b_i^{r*} follows from Proposition 4.2.

When offered this contract, hedge funds are indifferent. Hence, they choose to participate. Therefore, $\theta_i = 1$. □

Proof. (***Proposition 4.5)***
If the dealer bank has negative expected cash holdings in period 2, it goes bankrupt in period 1 without making any decision.

This happens when:

$$R^B\alpha^*E + \left(\left(\frac{R^H}{p}\left(1 - \frac{\lambda E - (1-\alpha^*)E}{Q_m}\right) - \frac{1-p}{p} - \frac{(U_H-1)B}{pQ_m}\right) - 1\right)Q_m < 0$$

Rearranging terms, the dealer bank goes bankrupt in period 1 if:

$$Q_m < \frac{R^H\left(\lambda - (1-\alpha^*) - pR^B\alpha^*\right)E + (U_H-1)B}{R^H - 1}$$

□

Proof. (***Proposition 6.1)***

Using Assumption (2), Lemma 4.1 and Proposition 4.2, the constraints to the dealer bank's problem in this case is:

The constraints in the problem of the dealer bank are:

$$\frac{R^H}{p} - \frac{1-p}{p(1-h)} - \frac{\tilde{U}_H}{p(1-h)} - 1 - r \geq 0 \qquad (\eta_1) \tag{17}$$

$$\lambda E \leq \left(1 - \alpha^* + \alpha^L\right)E + hQ_m \qquad (\eta_2) \tag{18}$$

$$0 \leq \alpha^L \leq 1 \qquad (\eta_3, \eta_4) \tag{19}$$

where $\eta_1, \eta_2, \eta_3 and \eta_4$ are the respective Lagrange multipliers.

The first order condition with respect to α^L in the problem in Section 6.1 is:

$$-R^B\alpha^*E + \eta_2\alpha^*lE + \eta_3 - \eta_4 = 0$$

From the first order condition with respect to h and using Lemma 4.1, I get:

$$\eta_2 = \frac{R^H}{p}$$

Hence,

$$\eta_3 - \eta_4 = \left(R^B - \frac{R^H}{p}l\right)\alpha^*E$$

If $l > p\frac{R^B}{R^H}$, then $\eta_4 > 0$ which implies that $\alpha^{L*} = 1$.

In that case, $h^* = h^{liq}_{min} = \frac{\lambda E - (1-\alpha^* + \alpha^* l)E}{Q_m}$. Equilibrium interest rate is found from hedge fund participation constraint (3) and b_i^{r*} follows from Proposition 4.2.

□

Proof. (***Proposition 6.2)***
From the first order condition of the dealer bank's problem with respect to α^L found in the proof of Proposition 6.1:

$$\eta_3 - \eta_4 = \left(R^B - \frac{R^H}{p} l \right) \alpha^* E$$

If $l \leq p \frac{R^B}{R^H}$, then $\eta_3 > 0$ which implies that $\alpha^{L*} = 0$.

In that case, $h^* = h_{min} = \frac{\lambda E - (1-\alpha^*)E}{Q_m}$. Equilibrium interest rate is found from hedge fund participation constraint (3) and b_i^{r*} follows from Proposition 4.2.

□

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