Macroeconomic Patterns in System-Wide Liquidity Regimes

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Abstract

We present a Bayesian estimation of hidden Markov chain (HMC) models to measure the latent structure of liquidity in the financial system. Starting with granular measures of market liquidity for corporate equities and bond markets and commodity futures markets, we use Markov chain Monte Carlo (MCMC) analysis to estimate the latent structure governing liquidity at the aggregate, system-wide level. Our input measures of market liquidity take advantage of recent work by Kyle and Obizhaeva [2011a,b], who have developed "invariant" price-impact measures of responses to order flow that are readily comparable across a broad range of financial markets and conditions. We find that three latent liquidity regimes—corresponding to high, medium, and low price-impact—are adequate to describe all of the markets we consider. In a more focused examination of the equities markets alone, we test the ability of several macroeconomic time series to recover the estimated liquidity dynamics. This exercise has significant explanatory power and allows for an economically meaningful attribution of the estimated latent liquidity states.

1 Introduction

This paper applies Bayesian estimation of hidden Markov chain (HMC) models to measure the structure of liquidity in the financial system. Starting with granular measures of market liquidity, we use Markov chain Monte Carlo (MCMC) analysis to estimate the latent structure which explains liquidity at the aggregate, system-wide level. Our input measures of market liquidity take advantage of recent work by Kyle and Obizhaeva [2011a], who have developed an "invariant" price-impact measure of responses to order flow that are readily comparable across a broad range of financial markets and conditions. We begin by exploring daily market data from a range of financial markets to create a collection of daily price impact measures. In our initial implementation, we consider volatility index futures, oil futures, and various portfolios of equities and corporate bonds. We assume that the dynamics of each of these daily price impact measures (33 in all) are determined by an underlying variable that alternates between one of several liquidity states and that switches between these states drives sudden changes in the observed levels of price impact. These underlying states are "latent"—i.e., not directly observable—but inferred from the dynamics of daily price impact measurements. In our initial analysis although we estimate each price impact series independently, assuming no coordination between the dynamics of the latent liquidity states across markets, we find surprising consistency in the dynamics of market liquidity across all of these markets. First, we find just three liquidity regimes are adequate to describe each market: high, intermediate, and low. Second, we find that the low liquidity regime afflicts all of the markets during the financial crisis of 2008.

Despite these common features, we also find interesting differences across the various markets in the lead-up to the recent crisis and in its aftermath. We build on our initial findings and on earlier studies that search for "commonalities in liquidity" (e.g., Chordia et al. [2000] and Karolyi et al. [2012]), by using a model that links a collection of latent

liquidity states (from multiple markets) together using a multinomial Probit model that is driven by a collection of macro variables. This provides a framework that allows us to asses the usefulness of macro variables as potential indicators of financial stress (at least in terms of market liquidity). Although we restrict our focus in this phase of the analysis to liquidity in the U.S. equities markets for reasons of data consistency, the model reveals that a number of macro variables, such as the Dow Jones Real Estate Index, the TED spread, the VIX® and the S&P 500 price-to-book (P/B) ratio are statistically significant with relation to explaining when these markets experience different levels of liquidity.

The remainder of the paper is structured as follows. After giving a description of issues related to measuring liquidity, we give describe the models and sampling strategies used in our MCMC analysis for both the univariate (one market at a time) models and the hierarchical (multiple markets at a time) model. Following the model description we describe the data and specific formula used to calculate the price impact measures used in the analysis. We report our findings from aggregating both the market specific analysis and the multiple market analysis. We conclude with a discussion of potential future work, limitations of the current approach and ways that these limitations could be overcome to allow these tools to be used to better understand and expand our ability to forecast liquidity.

1.1 The Challenges of Liquidity Measurement

Conceptually, "liquidity" is the ease with which participants in the financial system can convert their claims to cash. The settlement obligations of the vast majority of financial contracts are stated in terms of cash: payors must deliver cash, and payees must accept it. As a result, the ability of market participants to "get to cash" has important implications for the overall functioning of the system.

Liquidity is particularly important for the study of financial stability, as sudden shifts

in liquidity have historically been one of the defining characteristics of financial crises.[1] For a financial asset, liquidity arises because agents are ready and willing to purchase an instrument. In efficient markets, arbitrage implies that there is money to be made by those with the robust valuation models and accurate information to feed them, so that counterparties should be easy to find for a modest price concession. That is, liquidity providers will step in opportunistically if the asset is offered at an acceptable discount, but have little incentive to pay more than the current market price. The practical upshot for liquidity measurement is that we measure the extent of illiquidity in the system as one-sided deviations from an ideal benchmark of "perfect liquidity."[2]

Ideally, measures of liquidity to support financial stability monitoring would be both timely (available at high frequency to track developments in real time) and forward-looking (possessing some forecasting power to serve as an early warning signal). These goals are often defeated in practice by certain fundamental challenges. Liquidity exhibits three interrelated characteristics that present special complications to measurement: latency, nonlinearity, and endogeneity. Each of these challenges has ramifications for both funding liquidity and market liquidity.

[1] Kindleberger [1993, ch. 15], for example, recounts the history of crises in Western economies, with a special focus on the lender of last resort as a provider of backstop liquidity to the system. Reinhart and Rogoff [2009], and Schularick and Taylor [2012] consider a similar sample, with the latter focusing on the role of "credit booms gone wrong" as a precursor to crisis events. Because aggregate credit growth is typically facilitated by expanding bank balance sheets, there is an important empirical connection linking credit cycles, leverage cycles, and liquidity cycles.

[2] It is important to consider what constitutes a "reasonable" price in this context. Most financial instruments have limited liability, so there is typically some nonnegative price at which buyers come forward; however, a market is not liquid if buyers only appear for fire-sale offers. The techniques of financial engineering mean that plausible mark-to-model valuations are usually available even for contracts that trade infrequently. The recent Presidential Address to the Econometric Society by Holmström [2012] emphasizes the lengths markets will endure to achieve liquidity by forcing assets to be "informationally insensitive." See also Dang et al. [2012].

1.1.1 Latency

Latency means that much of the most interesting liquidity behavior is ex-ante unobservable. At the microstructural level, we typically most wish to know not merely the prices of recent trades or the current best bid and offer, but how deep or resilient the market(s) will be. Many trading mechanisms designed to attract liquidity providers (i.e., buyers and sellers) do so by restricting information availability, for example, with closed limit-order books, hidden or "iceberg" orders, anonymous brokerage to conceal trader identities, and limited-access "upstairs" trading venues for large trades. Moreover, traders are never under compulsion to reveal their intentions by actually issuing (or cancelling) a limit order prior to the moment of truth; some version of a market order is typically available. Conversely, private visibility into their own customers' positions and order flow can be a valuable information source for dealers.

To hedge against these and other liquidity surprises, firms frequently arrange for contingent liquidity in the form of lines of credit or derivative contracts. However, hedgers must still worry about the wrong-way risk that their supposed guarantors will themselves fail under the precisely the event being insured against. Liquidity is never purely localized to a single transaction or market. At the broadest levels, global liquidity depends in part on the responses of firms (and policymakers) to situations they may have not have planned for explicitly. For example, it is difficult to know in advance whether an initial deleveraging event will generate enough selling pressure to create a fire-sale feedback loop. Geanakoplos [2003], for example, emphasizes that collateral margins are likely to bind in a crisis, unexpectedly depriving participants of flexibility at the crucial moment. Similarly, the repeated clearing crises of the nineteenth century (see Calomiris and Gorton [1991]) were invisible to bankers in the system until it was too late. In general, liquidity measurement is "more honored in the breach," in the sense that it is easier to assess the depth or resilience of the market(s) under conditions stressful enough to violate the "perfectly liquid" ideal.

We address the challenge of latency by adopting a MCMC approach to estimate the latent regime structure governing the observed price impact series. The maintained assumption is that, while the markets' liquidity behavior is indeed largely latent, these hidden patterns will reveal themselves, given a broad cross-section of markets observed at relatively high-frequency (daily). In the results below, we are indeed able to identify three meaningful latent liquidity states (high, medium, and low price-impact) that seem to govern the observed liquidity behavior.

1.1.2 Nonlinearity

Nonlinearity in the response of liquidity to significant market changes compounds the problem of unobserved behavior. Numerous studies have documented the empirical regularity that price response to order flow tends to be concave function of the transaction size. Intuitively, order flow can move the price significantly before additional liquidity providers arrive to dampen the effect.[3] In contrast, recent work by Kyle and Obizhaeva [2011a,b] argues for a cube-root rule, and provides a theoretical justification for why this regularity should emerge. For example, limit orders (and other contingent liquidity) may crowd behind the best posted quote, so that an order flow impulse large enough to work through this initial phalanx may expose gaps in the book, provoking an abrupt shift in prices. Such unevenness in market depth may be an important source of fat-tailed returns distributions. Duffie [2010] suggests that for some relatively illiquid markets, it may be weeks before support arrives in the form of additional order flow. Nonlinearity is a challenge for liquidity measurement because it hampers our ability to extrapolate from small-scale, localized effects to the larger, out-of-sample effects that are often of greatest concern.

Nonlinearity in liquidity is an even greater worry for systemic stability, where the

[3] Much of the literature identifies a "square-root rule" that posits price impact to be proportional to the square root of the transaction size; see for example, Gabaix et al. [2006], Bouchaud et al. [2008], Hasbrouck and Seppi [2001], and Toth et al. [2011].

stakes are correspondingly higher. At this level, interactions among nodes in the system can conspire to produce self-amplifying feedback loops. Tirole [2011] provides a tour of the catalog of systemic pathologies related to illiquidity, including contagion, fire sales, and market freezes. He underscores the central fact that one of the basic services provided by the banking (and shadow banking) sector—namely maturity transformation—render it especially vulnerable to runs and other liquidity surprises. Brunnermeier [2009] provides a good overview of how these forces played out in practice, at least through the early (and most severe) phases of the recent crisis. He highlights four specific channels: (a) deleveraging spirals driven by erosion in capital levels and increases in lending standards and margins; (b) a credit crunch motivated by funding concerns; (c) outright runs, exemplified by Bear Stearns and Lehman Brothers; and (d) netting failures due to real or perceived counterparty credit risks. All of these modalities involve liquidity. Adrian and Shin [2010] emphasize the role played by institutional leverage in both the expansion and contraction of the system. Figure 1, adapted from similar illustrations in Adrian et al. [2013a], illustrates clearly that institiutional leverage expands and contracts via adjustments to assets and liabilities—not equity—suggesting that increases in leverage correspond to increases in overall liquidity, since bank deposits and other liabilities are a key component of liquid assets for other participants in the system. Leverage is strongly procyclical; by increasing global liquidity, aggregate balance sheet expansion encourages investment spending and tends to ease margin constraints. Figure 1 also indicates a correlation between bank leverage and market volatility, at least through the course of the most recent business cycle: periods of low volatility (green and yellow markers) tend to correspond to increases in bank leverage, and episodes of high volatility (orange and red) tend to match decreases in leverage. Unfortunately, the volatility-leverage-liquidity spiral works in reverse as well, leading to debt overhangs as the system contracts, with associated increases in institutional risk aversion and liquidity hoarding.

We address the challenge of non-linearity by agnostically allowing the data determine the "correct" number of liquidity states for each time series. Notably, for all 33 of our univariate series, three liquidity states are adequate to explain the observed variation in the price-impact statistics. Because the expected price impact is allowed to vary idiosyncratically for each of the three regimes (high, medium, and low price-impact), this model naturally captures non-linearities in price impact.

[Insert Figure 1 about here]

1.1.3 Endogeneity

Endogeneity means that liquidity is partly a network effect that emerges organically through the interactions of many market participants. Thus, active markets should have less need for identified liquidity providers who convert investments to cash by purchasing or rediscounting others' financial assets. Endogenous liquidity is a straightforward network externality in the sense of Pagano [1989] and Economides [1996]: investors are naturally more willing to enter markets where there are already many other traders and large transaction volumes, because this provides an implicit assurance that counterparties will be easy to find when needed. A familiar example of this phenomenon is the contrast between trading for on-the-run and off-the-run Treasuries (see Barclay et al. [2006]). Similarly, Bessembinder et al. [2006] find that liquidity externalities are consistent with the significantly reduced trade execution costs that followed the introduction of the TRACE feed, which increased transparency in the corporate bond market. Liquidity externalities are also often touted as a benefit of high-frequency trading.

Liquidity externalities operate at the level of the system as well.[4] They have long been a central concern of financial stability supervisors. Elliott et al. [2013], for example, doc-

[4]There are many discussion of this endogenous systemic externality. See, for example, Morris and Shin [2004], Dang et al. [2010], and Adrian and Shin [2010] and the references therein.

ument this multi-dimensional regulatory history for the U.S. One of the core functions of central banking is to provide a lender of last resort in a crisis, an idea first flirted with as an expedient in London's Panic of 1825, "codified" in Bagehot [1873], and institutionalized in the U.S. with the creation of the Federal Reserve in 1913. Recourse to a potentially unlimited source of liquid cash from outside the network of banks and financial firms is important in a crisis, when information asymmetries and other constraints prevent firms from liquidating their assets to meet withdrawals. On the other hand, some have argued that this explicit promise of effectively unlimited contingent liquidity creates a moral hazard—that too-big-to-fail banks undertake excessive leverage and maturity transformation, comfortable that the Fed's emergency backstop provides them with a free "liquidity put."[5] Moreover, in spite of discount-window access, Cornett et al. [2011] find that banks in the recent crisis were forced onto a more defensive liquidity posture, in part because Lehman Brothers' failure diverted commercial paper borrowers to draw on banks' liquid assets via backup lines of credit, and partly because wholesale funding sources suddenly shrank. A net result was a restriction in commercial lending. In the wake of the crisis, macroprudential supervisors have focused renewed attention on liquidity buffers, including the new Basel III requirements for banks' to maintain net stable funding and liquidity coverage ratios.[6]

We address endogeneity by estimating a hierarchical model that searches for common liquidty structure throughout the cross section of observed price-impact series. Although this work is in preliminary stages, and is currently limited to the cross-section of equity markets, we are able to identify significant patterns and attribute them statistically to

[5]Goodhart [2008] and Farhi and Tirole [2009] have made this argument. The term "liquidity put" is a metaphor for a commonly used recourse covenant that allows investors in a partially debt-funded structured investment vehicle (SIV) to put back their shares in the SIV to the sponsoring bank if the SIV is unable to roll over its short-term debt; see, for example, Entwistle and Beemer [2008]. However, during at least one episode—the Y2K millennium date change—the Fed literally sold liquidity put options; see Sundaresan and Wang [2006].

[6]The new liquidity framework for banks is discussed in Basel Committee on Banking Supervision [2013, 2010], Adrian et al. [2013b], and Bank of England [2011].

particular macroeconomic time series, providing some economic interpretation for the estimation.

1.2 Liquidity Measurement in Practice

Research contributions generally focus either on liquidity in the narrow context of a particular financial market—so-called "market" or "microstructural" liquidity—or at the aggregate level of the financial system as a whole—so-called "global" or "funding" liquidity. How one measures liquidity depends in part on the portion of the financial system under examination. Brunnermeier and Pedersen [2009] connect these two strands of the literature, using bank balance sheets as an organizing device, as depicted in Figure 2. In this framework, the distinction between "market liquidity" and "funding liquidity" is based essentially on which side of a financial firm's balance sheet is involved. Market liquidity refers to the ease with which financial institutions can convert securities or loans from their asset portfolio to cash.[7] Funding liquidity refers to the ease with which institutions can obtain cash by borrowing in funding markets. Figure 2 underscores that one of the central functions of banks and similar intermediaries is to convert relatively long-maturity, low-liquidity commitments on the asset side to relatively short-maturity, high-liquidity obligations on the liability side of intermediaries' balance sheets. Official liquidity represents the range of short-term cash resources available in a financial crisis—that is, when the wholesale funding markets fail—from central banks and other agencies, enterprises, and programs with explicit or implicit taxpayer backing.[8] Because these liquidity resources

[7]Harking back to Moulton [1918], Mehrling [2010] refers to this sort of liquidity as the "shiftability" of bank or dealer assets—i.e., the ability to shift them into cash. It is worth noting that the asset side of Figure 2 represents a primary point of contact between the financial system and the real economy. Commitments like corporate debt or mortgage loans typically translate directly into real activity such as workforce expansions and home improvements investments. Cornett et al. [2011] analyze market and funding liquidity empirically, along with their net effect on overall credit supply.

[8]In the U.S., these resources include deposit insurance, the Fed's discount window, Federal Home Loan Bank advances, as well as the numerous emergency facilities created as expedients in the recent crisis. Official liquidity is sometimes called "outside" liquidity, meaning it comes from outside the markets

typically come into play only in unusual but important occasions, they appear as dashed arrows in the figure, which flow in only one direction.

[Insert Figure 2 about here]

1.2.1 Aggregate Liquidity Measures

Financial institutions can participate as both suppliers and demanders of liquidity in these markets, as indicated by the bidirectional arrows. An institution's market liquidity needs are typically handled by drawing on its own cash reserves or by selling assets for cash. If the ordinary give and take of trading does not net out, the firm can turn to the funding markets to borrow or lend the difference. Wholesale funding markets thus aggregate much of the endogenous net supply and demand of liquidity overall, and prices in these markets provide a bellwether for the state of system. Figure 3 depicts several commonly used measures of aggregate liquidity conditions derived from prices in wholesale funding markets. Following Brunnermeier [2009], Boudt et al. [2013], and Boyson et al. [2010], we proxy the Treasury-Eurodollar (TED) spread as the difference between 3-month T-bill yields and 3-month LIBOR.[9] A frequently cited alternate spread measure for funding liquidity conditions is the LIBOR-OIS (London interbank offered rate vs. overnight index swap) spread; see Gefang et al. [2011], Michaud and Upper [2008], and Taylor and Williams [2009]. Both spreads capture deviations of borrowing conditions in the interbank markets from "pure" credit-risk-free borrowing, and thus reflect a variety of sources of reticence to lend to banks, including credit risk and aggregate liquidity anomalies. The $VIX^{®}$ is a traded index of market volatility. Its gradual downward drift during the pre-crisis period (roughly 2002 to 2007) is symptomatic of the so-called "volatility paradox": market risk as measured

themselves. For a definition and model of inside and outside liquidity, see Holmström and Tirole [2013].

[9]Futures traders at the Chicago Mercantile Exchange first noted the TED spread in the early 1980s, where they tracked the pricing differential between the 3-month T-bill futures and 3-month Eurodollar futures contracts, which traded in neighboring pits McCauley [2001].

by price volatility was dropping while overall risk exposures were simultaneously (and not coincidentally) building across the system (Brunnermeier and Sannikov [2012]).

[Insert Figure 3 about here]

The first major foreshock of the crisis came in August 2007, triggered by an absence of liquidity that prevented BNP-Paribas from marking to market several of its investment funds backed by subprime mortgages. The surprise provokes a sharp but very temporary drop in the 3-month repo rate and a simultaneous more permanent jump in both the TED and LIBOR-OIS spreads. Market turmoil persists through the failure of Bear Stearns in March 2008, until the failure of Lehman Brothers in September 2008 raises liquidity problems to a new level. Interest rate spreads provide a high-frequency glimpse into aggregate liquidity conditions, but price moves can only hint at the underlying changes in cash holdings. As financial firms withdrew, the aggregate endogenous liquidity supply in the wholesale funding markets became inadequate to satisfy current cash obligations, and official liquidity providers were forced to step in. The Federal Home Loan Banks (FHLBs), provided a first line of defense, via advance funding to member institutions, which include most large commercial banks (Ashcraft et al. [2010]. Over the first few months after August 2007, FHLB advance funding increased by more than $200 billion. Although this recourse to official liquidity was modest compared to what was to come, it was a significant departure from business as usual at the time. The full-blown crisis emerges with the Lehman failure in September 2008. Wholesale markets collapsed, and financial firms proceeded en masse to the Fed's backstop liquidity programs.[10] Surprisingly, the Fed's sig-

[10]The Bear Stearns failure necessitated recourse to the Fed, including the creation of two brand new liquidity vehicles, the Primary Dealer Credit Facility (PDCF) and the Term Securities Lending Facility (TSLF). Following the Lehman failure, the brunt of the wave of new funding demands was borne initially by the Fed's swaps facility, along with FHLB advances, the PDCF and TSLF again, and the newly created ABCP MMMF liquidity facility (AMLF). Much of this funding subsequently transitioned to other programs, including the Treasury's new Troubled Asset Relief Program (TARP), created in October 2008. For an analysis of the Fed's various large-scale asset purchase programs, see Chen et al. [2012a], D'Amico and King [2012], and D'Amico et al. [2012].

nature lender-of-last-resort facility, the discount window, plays a miniscule role throughout the episode.

The sharp shift in supply and demand for wholesale liquidity also shows up in the management of banks' cash reserves, as depicted in Figure 4. Prior to the Lehman shock, aggregate bank reserve balances (which paid no interest prior to July 2013) hovered near zero; banks continued to rely on wholesale funding markets to meet short-run cash contingencies. After the Lehman failure, banks begin to hold precautionary reserve balances in significant quantities; this practice of reserving has continued essentially unabated. At the same time, the Federal Reserve has flooded markets with liquidity, driving yields on overnight Fed funds and T-bills to near zero. The persistence of the short-term riskless rate near the zero lower bound while loanable funds pool up—potential lenders always have the alternative of holding cash instead of accepting a negative return—suggests a market failure.

[Insert Figure 4 about here]

1.2.2 Granular Liquidity Measurement

At the aggregate level of funding liquidity, a primary concern is whether the financial system has the internal flexibility to satisfy all of its immediate funding needs. Economic equilibrium means that every borrower in the wholesale funding markets should be able to find a willing lender, but the long and painful history of systemic crises demonstrates that this equilibrium is not reliable. This is the purview of central banking, macroprudential regulation, and systemic supervision. At the other extreme, market liquidity focuses on the intricacies of specific markets. The availability of granular, high-frequency data on transaction prices, bid-ask quotes, trading volumes and customer order flows facilitates detailed modeling of the behavior of market participants. Market liquidity is typically measured by

the price concession required to unload (or acquire) a given investment position in a very short time interval.

By design, liquidity measures based on prices and volumes in the wholesale funding markets aggregate information from across the financial system. There are only a relative handful of funding products traded in these markets, which in turn are dominated by a relatively small set of very large institutions. Yet the issue of liquidity also applies to the many thousands of markets for equities, bonds, indexes, commodities, and derivatives. Although these smaller markets are not as immediately connected to system-level stability, they are much more numerous than the interbank funding markets. Liquidity in these markets may therefore carry additional information about overall liquidity that is lost in the aggregation to the wholesale level. In particular, asset markets offer a smorgasbord of different industries, product types, geographic concentrations, maturity habitats and credit grades that is not available in the short-term, interbank funding markets. It is an empirical question whether this cross-sectional diversity is apparent in liquidity, and whether it this cross-sectional information is helpful in understanding systemic behavior.

This work builds on earlier studies that look for aggregate liquidity patterns. The article by Chordia et al. [2000] was the first in a series of papers that search for "commonalities in liquidity" in the cross section of equity markets. Karolyi et al. [2012] is a recent installment, extending the analysis to an international comparison of thousands of stocks in 40 countries. Commonalities in liquidity exist, and unsurprisingly differ significantly across countries and over time.[11] Recent research by Chen et al. [2012b] combines price information from financial markets with quarterly quantity information from the Flow of Funds data in an effort to distinguish the differential impact of shifts in liquidity demand

[11] Commonality is measured by the R^2 of each stock's daily price impact measure (see Amihud, 2000) on the average price impact for all other stocks in the country. Individual stock commonality measures are averaged to get a country-level commonality index. Karolyi et al. [2012, p. 99] attribute the time-series variation to both supply- and demand-side proxies in funding markets via regression analysis, noting that "demand-side explanations are more reliably significant."

vs. liquidity supply. They distinguish between "core" and "noncore" liquidity, with the primary difference arising from the inclusion in noncore of financial firms' liabilities held by other financial institutions. Our research efforts in this area differ in scope from these earlier studies. The goal is to exploit fine-grained information—such as daily data on prices and volumes—from a wide range of asset markets, including equities, bonds, and commodities. By casting a wide net across many diverse instrument types, the approach should have a better chance of detecting emerging liquidity anomalies and identifying key liquidity indicators and important patterns among the markets being monitored.

Our analysis exploits recent work by Kyle and Obizhaeva [2011b,a] on generalized "price impact" measures of market liquidity. Price impact refers to the change in market prices caused by a one-directional order flow (buy or sell) of a given size.[12] For example, it is always a challenge for an investor or dealer to sell a large block of stock, but especially so in stressful market conditions when crowded trades can conspire to generate fire-sale price drops. By design, price impact measures reveal the non-linearity of market depth in the face of transaction pressure. Kyle and Obizhaeva posit a scaling of trading activity by "business time" that allows for the construction of price impact measures that are invariant to both the institutional details of diverse market microstructures as well as the pace of trading activity. This invariant approach facilitates a systematic analysis by allowing the repeated application of the same calculation methodology across a range of markets.

2 Model Description

The primary assumption underlying our analysis is that the liquidity in a specific financial market (as defined by a portfolio of publicly traded securities) switches between distinct states, jumping from one state to another (e.g., low to high liquidity) and then staying

[12] While a number of alternative metrics exist (e.g. big ask spreads, price impact measures, trading volumes), these tools are typically restricted or customized to a single market.

that new state for a random period of time. The subsequent observed level of liquidity is a random deviation from the level related to the average liquidity for the current state. While the observed liquidity sometimes falls between the average liquidity for two different states, ultimately it is the persistence of the observed liquidity that identifies an underlying state. When liquidity from a multiple financial markets is considered, we augment the liquidity models for each individual market with an 'add on' hierarchical model which can explain, in part, periods of coordination where a large subset of the financial markets exhibit similar liquidity patterns. The hierarchical portion of the model, which links individual liquidity models together, can be considered an 'add on' because it does not feedback into the individual level liquidity models. Instead it allows us to determine whether macro variables based on economic summaries of the broader financial markets and economy are related to liquidity states across multiple markets. Uncovering such a relationship offers a framework for understanding and potentially predicting (by predicting the underlying dynamics of the macro variables) when system-wide liquidity stress might occur.

We start by proposing a univariate, hidden Markov chain model for each financial market where liquidity is a random deviation from a latent value associated with each state of the hidden Markov chain. We consider two variations of random deviations, independent deviations around an average level and deviations around a value that mean reverts around an average level. Initially we assume that the dynamics of these models are unrelated, then we propose a hierarchical (multiple market) model where macro variables are used to explain the states identified by the collection of univariate hidden Markov models.

2.1 Univariate Models

Liquidity measurement over T periods, $y_i = (y_{i1}, ..., y_{iT})^T$, for market i are assumed to be a random normal deviation around a dynamic, latent level of liquidity $\theta_i = (\theta_{i1}, ..., \theta_{iT})$, or

$$y_i = \theta_i + \epsilon_i,$$

where

$$\epsilon_i \sim N(0, \sigma_i^2 I_T)$$

and I_T is a T dimensional identity matrix. For the first version of the model, the hidden Markov chain (HMC) only version, the latent level θ_i is one of K levels, each of which represents a different level of liquidity or state for each market specific, discrete-time hidden Markov chain D_i, or

$$\theta_i = F_i \bar{\theta}_i,$$

where $\bar{\theta}_i$ is a $K \times 1$ vector and each element represents the average level of liquidity for the kth state of D_i, or

$$F_i(t, k) = I\{D_{it} = k\}$$

and $I\{\}$ is an indicator function equaling either 0 or 1.

For the second version of the model, the Mean Reverting, hidden Markov chain (MRHMC) version, the latent level θ_i mean reverts around the average level associated with the state

of D_i, or for $t = 2, ..., T$,

$$\Delta\theta_i = \gamma_i((\theta_i)_{-T} - (F_i\bar{\theta}_i)_{-1}) + (\xi_i)_{-1}, \tag{1}$$

where $()_{-1}$ indicates that the first element and $()_{-T}$ indicates that the last element of the vector () has been removed, and

$$\Delta\theta_{it} = \theta_{it} - \theta_{it-1}.$$

For $t = 1$ we let

$$\gamma_i\theta_{i1} = \gamma_i\bar{\theta}_{i1} + \xi_{i1},$$

where

$$\xi_i \sim N(0, w_i I_T).$$

We require $0 < \gamma_i \leq 1$, which ensures that θ_i is stationary and increases the variance of θ_{i1}, allowing the starting value of θ_i to be relatively vague. Alternatively, (1) can be rewritten as,

$$L_i\theta_i = \gamma_i F\bar{\theta}_i + \xi_i, \tag{2}$$

where L_i is a sparse $T \times T$ matrix with zeros except for the following elements, $L_i(j, j) = 1$ and $L_i(j, j-1) = \gamma_i - 1$ for $j > 1$ and $L_i(1, 1) = \gamma_i$.

For both versions of the model, the dynamics of D_i are given by an initial probability density ν_i, a $K \times 1$ vector, and a transition probability density P_i, a $K \times K$ matrix. Given

a realization of D_i, its density is given by

$$f(D_i) = \nu(D_{i0}) \prod_{t=1}^{T} P_i(D_{it-1}, D_{it}).$$

We assume conjugate priors for σ_i^2 and w_i (inverted Gamma), ν_i and each row of P_i (Dirichlet) and $\bar{\theta}_i$ and γ_i (truncated Normal). In addition, we use subjective priors based on initial conditional maximum likelihood estimates of summaries of the data, in order to ensure that the filtered hidden Markov chain model is able to clearly define between the dynamics of the hidden Markov chain and the dynamics of the latent value θ_i.

Full Conditional Distributions HMC Model

We use Markov chain Monte Carlo (MCMC) analysis to infer parameter values for both of these univariate models and multivariate model built on these univariate models. For a description of MCMC methods see Brooks, Gelman, Jones and Meng (2011) and Gelman, Carlin, Stern and Rubin (2000). The full conditional densities used in the MCMC analysis for the HMC model are as follows:

$$\bar{\theta}_i|- \sim N\left(\left(\frac{1}{\sigma_i^2}F_i^T F_i + \frac{1}{\tau_{\bar{\theta}_i}^2} I_K\right)^{-1}\left(\frac{1}{\sigma_i^2}F_i^T y_i + \frac{\mu_{\bar{\theta}_i}}{\tau_{\bar{\theta}_i}^2}\bar{\bar{\theta}}_i\right), \left(\frac{1}{\sigma_i^2}F_i^T F_i + \frac{1}{\tau_{\bar{\theta}_i}} I_K\right)^{-1}\right) I\{\bar{\theta}_{i1} < ... < \bar{\theta}_{iK}\};$$

were $-$ represents everything else remaining in the model and

$$\frac{1}{\sigma_i^2}|- \sim Gamma\left(shape_{\sigma_i^2} + \frac{T}{2}, scale_{\sigma_i^2} + \frac{1}{2}(y_i - F_i\bar{\theta}_i)^T(y_i - F_i\bar{\theta}_i)\right).$$

Realizations of the hidden Markov chain D_i, conditional on the remaining parameters and Data, are generated following the Filter Forward, Sample Backwards approach commonly used with discrete-time Hidden Markov chains, see Bahm, Petrie, Soules and Weiss (1970) and described in a more general continuous-time framework in Cappe, Moulines

and Ryden (2005). For completeness the filter forward equations for the HMC model are given bellow,

$$f\left(y_{it}|-,\mathcal{F}_{it-1}\right)=\sum_{k=1}^{K}f\left(y_{it}|D_{it}=k,-,\mathcal{F}_{it-1}\right)f\left(D_{it}=k|-,\mathcal{F}_{it-1}\right), \tag{3}$$

were $\mathcal{F}_{it}=\{Y_{i1},...,Y_{it}\}$ and by

$$f\left(D_{it}=k|-,\mathcal{F}_{it}\right)=\frac{f\left(y_{it}|D_{it}=k,-,\mathcal{F}_{it-1}\right)f\left(D_{it}=k|-,\mathcal{F}_{it-1}\right)}{f\left(y_{it}|-,\mathcal{F}_{it-1}\right)}. \tag{4}$$

Specifying a vague initial state probability, e.g.,

$$f(D_{i0}=k|-,\mathcal{F}_{i0})=\frac{1}{K},$$

completes the forward recursion. The key equation for the backward sampling is given by the density of the hidden Markov chain, conditional on all of the data, or

$$\begin{aligned}&f\left(D_{iT-t}=k|-,\mathcal{F}_{iT}\right)=\\&\textstyle\sum_{j=1}^{K}\frac{f(D_{iT-t+1}=j|D_{iT-t}=k,-,\mathcal{F}_{iT-t})f(D_{iT-t}=k|-,\mathcal{F}_{iT-t})}{f(D_{iT-t+1}=j|-,\mathcal{F}_{iT-t})}f\left(D_{iT-t+1}=j|-,\mathcal{F}_{iT-t+1}\right).\end{aligned} \tag{5}$$

Given these formula, generating a realization is straightforward: i) calculate the forward filter; ii) generate a sample for D_{iT} from (4), with $t=T$; and iii) recursively calculate $f\left(D_{iT-t}=k|-,\mathcal{F}_{iT}\right)$, conditional on all of the draws $(D_{iT},...,D_{iT-t+1})$ using (5) and use this to generate a sample for D_{iT-t}. Given a realization of D_i the full conditional distribution for each row of the transition probability is given by

$$P_i(j,:)|-\sim Dirichlet\left(\alpha_{i1}+n_{ij1},...,\alpha_{iK}+n_{ijK}\right),$$

where α_{ijk} is the prior associated with D_i jumping from state j to k and n_{ijk} is the actual number of times that the current realization of D_i jumps from state j to state k. A similar full conditional density exists for ν_i, but this is inconsequential as the backwards recursion dominates the *a priori* initial state. It is important to note that the MRHMC model is disentangling two dynamics, the dynamics of D_i and θ_i. In practice we found that the model required a strong priors on the dynamics of D_i in order to obtain a meaningful distinctions between these two dynamics. Setting α_{ikk} to a sufficiently large value, suggesting *a priori* that the hidden chain is persistent results in a clean separation of these two competing dynamics.

Full Conditional Distributions MRHMC Model

There are similarities between some of the full conditional densities of the MRHMC model and the HMC model. The full conditional density for P_i is unchanged, while the full conditional density for $\bar{\theta}_i$ is obtained by replacing y_i with $\frac{1}{\gamma_i}L_i\theta_i$ and σ_i^2 with $\frac{w_i}{\gamma_i^2}$. The full conditional density for $\frac{1}{\sigma_i^2}$ is obtained by replacing $F_i\bar{\theta}_i$ with θ_i and the full conditional density for D_i is obtained by replacing the likelihood $f\left(y_{it}|D_{it}=k,-,\mathcal{F}_{it-1}\right)$ used in (3) and (4) with $f\left(\theta_{it}|D_{it}=k,-,\theta_{it-1},...,\theta_{i1}\right)$. The remaining full conditional densities for the MRHMC model are as follows:

$$\frac{1}{w_i}|- \sim Gamma\left(shape_{w_i}+\frac{T}{2}, scale_{w_i}+\frac{1}{2}(L_i\theta_i-\gamma_iF_i\bar{\theta}_i)^T(L_i\theta_i-\gamma_iF_i\bar{\theta}_i)\right);$$

$$\gamma_i|- \sim N\left(\Sigma_i\left(\frac{1}{w_i}A_i^T\Delta\theta_i+\frac{\mu_{\gamma_i}}{\tau_{\gamma_i}^2}\right),\Sigma_i\right)I\{0<\gamma_i\leq 1\};$$

where

$$\Sigma_i = \left(\frac{1}{w_i} \left(A_i^T A_i + (\theta_{i1} - \bar{\theta}_{iD_{i1}})^2 \right) + \frac{1}{\tau_{\gamma_i}^2} \right)^{-1} \text{ and } A_i = \left((\theta_i)_{-T} - (F_i \bar{\theta}_i)_{-1} \right),$$

and

$$\theta_i | - \sim N \left(\left(\frac{1}{w_i} B_i + \frac{1}{\sigma_i^2} I_K \right)^{-1} \left(\frac{\gamma_i}{w_i} B_i \left(L_i^{-1} F_i \bar{\theta}_i \right) + \frac{1}{\sigma_i^2} y_i \right), \left(\frac{1}{w_i} B_i + \frac{1}{\sigma_i^2} I_K \right)^{-1} \right),$$

where B_i is a $T \times T$ matrix given by

$$B_i = \left(L_i^{-1} (L_i^{-1})^T \right)^{-1}.$$

An alternate approach for sampling θ_i and γ_i, conditional on D_i, is to treat them as a discrete-time, dynamic linear model and use a filter forward sample, backwards strategy like the Kalman Filter, see Kalman (1960) for the original reference and Carter and Kohn (1994) and DeJong and Shephard (1995) for MCMC based inference approaches. Although we explored a filter forward, backwards sample approach, we found that this approach was not as stable as the regression based approach detailed above. Obviously one disadvantage of the regression approach is the need to calculate B_i, which requires the inversion of a $T \times T$ matrix, something that can become computationally prohibitive as T becomes large. Fortunately, the form of L_i results in a banded matrix for B_i where every element, except for the diagonal and the rows just next to the diagonal, are zeros. In addition, the non-zero elements are functions of γ_i; to be explicit

$$B_i(j,j) = \begin{cases} 1, & \text{if } j = T \\ \frac{1}{2} + 2\left(\frac{1}{2} - \gamma_i\right)^2, & \text{otherwise} \end{cases}$$

and

$$B_i(j, j-1) = B_i(j-1, j) = \gamma_i - 1.$$

2.2 Hierarchical Model

The hierarchical "add on" model runs a collection, N, HMC or MRHMC models in parallel, one for each market being considered. At each sweep of the MCMC algorithm, a realization of the latent hidden Markov chain, for each market, is generated resulting in the collection of realizations $(D_1), ..., (D_N)$. For each realization every individual time point can be viewed as a draw from a multinomial distribution that is driven by a set of time-varying covariates x_t – the macro variables. Conditional on a current realization of the hidden Markov chain, the "add on" portion of the model is a multinomial Probit model, where

$$f\left(D_{it} = k\right) = f\left(\tilde{z}_{itk} > \tilde{z}_{itl}, l \neq k\right),$$

and $\tilde{z}_{it}$ is multivariate normal or

$$\tilde{z}_{it} \sim N\left(\tilde{\beta} x_t, \tilde{\Sigma}\right).$$

We follow McCulloch and Rossi's (1994) approach, which builds on Albert and Chib (1993), for dealing with the identification issues that arise in using a Bayesian approach for the multinomial Probit model. A related, alternative approach is discussed in McCulloch, Polson and Rossi (1998). The additive identification is overcome by forcing the latent value for state 1 to always be zero. This is done by defining z_{it} as follows,

$$z_{itk} = \tilde{z}_{itk} - \tilde{z}_{it1},$$

which results in (6) becoming

$$f\left(D_{it}=k\right)=\begin{cases} f\left(0>z_{itl}l\neq k\right), & \text{if } k=1 \\ f\left(z_{itk}>\max(0,z_{itl}),l\neq k,l>1\right), & \text{if } k>1 \end{cases}$$

where

$$z_{it}\sim N\left(\beta x_t,\Sigma\right).$$

and β is a $(K-1)\times p$ matrix, where p is the number of macro variables, including an intercept. The scale identification is overcome by restricting

$$\Sigma_{1,1}=1.$$

We assume conjugate priors for β and Σ and following McCulloch and Rossi (1994) we sample β and Σ from the unconstrained full conditional densities using Gibb Samplers and then rescale by dividing these draws by $\Sigma_{1,1}$, which enforces the above constraint.

The hierarchical portion of the model is considered to be an "add on" to the model because the distribution of the hidden Markov chains $(D)_1,...,(D)_N$ does not depend on the Multinomial Probit probabilities or more to the point they do not depend on the macro variables. In order for the distribution of the hidden Markov chains to depend on the macro variables, we would need to model the transition between the latent liquidity states (as opposed to modeling the states themselves) as multinomial random variables conditional on the macro variables, which is something that were are leaving for future research. Instead the current "add on" model summarizes the relationship between latent states and the macro variables, acting as a supplemental analysis that gives us insights into how the latent liquidity states related to the macro variables, but makes no assumption about nor

gives any insights into how the macro variables impact the dynamics of the latent states. A less sophisticated approach to getting the same insights would be to save a realization of each hidden Markov chain, from the MCMC analysis, calibrate a multinomial Probit model for this collection of realizations. Repeat this multiple times, each time with a different set of realizations obtained by stopping the MCMC analysis at a random time, which would result in a set of multinomial Probit parameter estimates, one for each set of realizations, and then average the parameters estimates from all of these analysis. Our approach is more elegant as it updates the parameters of the multinomial Probit model with each sweep of the MCMC analysis. The basis of the relationship between the hidden states and the macro variables is determined by the portions of the hidden Markov chains which are relatively stable (have a high probability of being in one of the states), which holds for large portions of time over each of the markets that we are considering. Portions of the hidden Markov chain that tend to switch states (have a probability that is distributed between two or more states) have less impact as the hidden Markov chains alternate between these competing states during the analysis and require estimates of β, which can reasonably accommodate this oscillation.

The fact that the state of a hidden Markov chain can switch states during the analysis presents a technical challenge. When the hidden chain changes state, the latent values from the Multinomial Probit model, the z_{it} have to change to match their likelihood function (e.g., assume that chain i at time t changes from state 2 to state 3, then z_{it3} must become positive and z_{it2} must be less than z_{it3}). From a practical standpoint, we found that when a hidden Markov chain changes state, we can sample from the truncated, full conditional density of each latent variables in order to impose the new ordering, but doing this once is typically not sufficient to provide a stability for the estimate of β and Σ. This stability issues can be overcome by drawing a small number (on the order of a few dozen) samples of all of the related latent variables (e.g., draw repeatedly from z_{it} when a new ordering

constraint imposed by the change in state).

3 Data

We compute the Kyle-Obizhaeva price-impact on a daily basis across 33 markets in total, covering thousands of individual securities in four distinct asset classes. By casting a wide net across all of these diverse securities, this approach has a greater chance of identifying emerging risks in liquidity and discerning important patterns among the markets being monitored. Specifically, our initial dataset includes the following instruments:

- all U.S. equities from the Center for Research in Security Prices (CRSP) which maintains a comprehensive collection of security price, return, and volume data for the NYSE, AMEX, and NASDAQ stock markets,
- all U.S. corporate bonds from the Trade Reporting and Compliance Engine (TRACE), which is FINRA's over-the-counter corporate bond market real-time price dissemination service; introduced in 2002, TRACE consolidates transaction data for all eligible corporate bonds, which include investment grade and high yield debt,
- West Texas Intermediate (WTI) light sweet crude oil futures from the New York Mercantile Exchange, which is the world's largest-volume futures contract traded on a physical commodity; data for contracts with expiration in one-month out to six-months were collected from Bloomberg, and
- S&P 500 market volatility index ($VIX^{®}$) futures from the Chicago Board Options Exchange, which is a pure play on implied volatility designed to reflect investors' view of future (30-day) expected stock market volatility; introduced in 2004, data for contracts with expiration in one-month out to nine-months were collected from Bloomberg.

The construction of the liquidity measures starts with 2004, when all series are available, and for each security the daily price and volume information were used to calculate the daily price-impact measures. Both the CRSP equities and TRACE corporate bonds data were grouped into portfolios based on one-digit Standard Industry Classification (SIC) codes, and a market-weighted price index is created for the collection of ten industry level stock and bond portfolios. For the SIC level portfolios, the analysis covers SIC 0 through SIC 8. The portfolio for SIC 9 proved to contain a minimal amount of individual stocks and bonds, and was described as a miscellaneous industry category. Because of this, SIC 9 did not produce meaningful results and was dropped from the analysis for both TRACE and CRSP.

VIX® and WTI futures were grouped by their maturity date. For the VIX® futures, this meant we had nine different securities from the front-month out to nine months forward. For the WTI futures, we had six different securities from the front-month out to six months forward. For both of these contracts, and for the futures market in general, the near-dated contracts are usually more actively traded than the longer-maturity futures. This is our reasoning behind only gathering data for the first month's forward, as there are many days of inactivity or no trading in contracts that mature far out in the future.

4 Liquidity Regimes

In our initial analysis we estimated each price impact series independently, using both the hidden Markov chain (HMC) and the mean reverting hidden Markov chain (MRHMC) models. Although there is no coordination between the dynamics of the latent liquidity states across markets for this initial analysis, we find surprising consistency in the dynamics of market liquidity across all of these markets. Despite common features, we find interesting differences across the various markets in the lead-up to the recent crisis and in its aftermath.

We formally explore these difference using the hierarchical model, which allows us to link the latent liquidity states (from multiple markets) together with a collection of macro variables. This provides a framework that allows us to asses the value of macro variables.

4.1 Individual Market Liquidity

We start by considering the performance of the two competing univariate models and provide evidence for our finding that there are essentially three different liquidity regimes across these different markets; then we report aggregate summaries based on these models.

Performance of Models

We found that both models could identify interesting liquidity regimes within the Kyle-Obizhaeva price-impact data over the range of different markets that we considered. The HMC only model is a simpler model and can readily identify the three liquidity regimes, see Figure 5; however, because of the outliers and the dispersion of the price-impacts, standard Bayesian model choice tools (which are discussed in the Appendix) support adding an increasing number of latent states (typically in excess of 10 to 15, even with heavy penalties for increased model complexity). A visual inspection of these higher state models indicates that increasing the number of states simply increases the number of states in the middle of the price-impact range, essentially breaking up the mid-liquidity state into a large number of sub-states.

[Insert Figure 5 about here]

We felt that it was important to have a parsimonious model, with respect to the number of states, and as a result developed the MRHMC model. As can be see in Figure 6, the mean-reverting portion of the model (in red) accommodates the outliers and dispersion of the price-impacts, such that a substantial clustering of points must be away from an

existing level before the model choice tools support adding an additional state. Given the parsimonious nature of the MRHMC model with respect to identifying the number of underlying states, we adopted it in favor of the HMC model.

[Insert Figure 6 about here]

While the Bayesian model choice approach is detailed in the appendix, it is worth pointing out some of the key parts of this approach and our primary finding that there are essentially three liquidity regimes across the wide range of markets that we examined. We placed a prior on the space of possible models for each market (where a model equates to a fixed number of liquidity regimes) so that increasingly complex models were *a prior* less likely. Typically we considered a fixed number of models (e.g., $K = 2, ..., 6$) and the prior for each model or value of K was a Geometric distribution in terms of the square of the number of models,

$$f\left(K = k\right) = \frac{\delta^{k^2}}{\prod_{j=1}^{K_T} \delta^{j^2}},$$

where K_T is the maximum number of hidden states considered and δ is a value set by the researcher. We ran a study to choose an appropriate level of δ , by first obtaining parameter estimates from a 3 state model using real data and then generating a collection of syntectic data sets using these parameters and restricting the number of states to be 3. We then choose values of δ that were most likely to recover the true model for these synthetic data sets (this process is discussed in more detail in the Appendix). Table 1 contains the posterior probability of the number of hidden states for all of the 33 markets that we considered.

[Insert Table 1 about here]

General Findings

One of the benefits of analyzing detailed and high-frequency data with just the univariate models, based on the Kyle-Obizhaeva price-impact data, is that it allows for a visual identification of basic cross-sectional liquidity patterns. This is valuable in developing research hypotheses, e.g., for the hierarchical model, or for general situational awareness in a policy context. Because the Kyle-Obizhaeva price impact values are scaled to be consistent across markets, we can provide a graphical summary of the price impact measure over time, Figure 7 shows the daily median value of the price impact measure across all 33 series, which will typically represent a different market each day, along with the daily interquartile range (i.e., the portion of the cross-sectional distribution between the 25th and 75th percentiles).

[Insert Figure 7 about here]

For comparison with more familiar liquidity measures, Figure7 also reproduces the TED spread. Reassuringly, we see that the price impact measures respond together with the TED spread to the large illiquidity events of the crisis, such as the run on the repo triggered by the BNP Paribas episode, and the failures of Bear Stearns and Lehman Brothers. The price-impact measures offer a couple of advantages over the TED spread, however. First, we are able to measure price impact individually across a wide range of markets. When aggregate illiquidity spikes, this collection of metrics facilitates an attribution to specific subsectors of the financial system. Second, the framework provides a panel of daily liquidity data to support a systematic investigation of reliable patterns in illiquidity across markets. Such patterns may even provide some forecasting power to support early warning systems. Because these data are measured in a regular and consistent way, they are amenable to well understood statistical tools.

Although the summary of the price-impact offers some insight, we find that in practice there is enough variation in these measure across the markets that a direct comparison

of the levels from each market is helpful. The estimated states, however, of the hidden Markov chains from the univariate MRHMC model, is invariant to differences in the price levels across markets and can clearly identify varying levels of liquidity. We labeled these states the: (1) low, (2) intermediate, and (3) high price-impact states for each series, where high price impact means low liquidity, and vice versa. This analysis resulted in a daily estimate of the probability that each market was in each of these three unobserved. Figure 8 presents the cross-sectional averages across the 33 series of these three probabilities, which must add up to one. Red indicates the likelihood of high price impact, and blue indicates low price impact; yellow is the intermediate state.

[Insert Figure 8 about here]

Although there was diversity in market liquidity for these 33 series, there were periods of common behavior. For example, the run on the repo market in August 2007 corresponds to a deep but short-lived spike in the probability of the low-liquidity (high price-impact) state. Similarly, the liquidity crisis after the failure of Lehman Brothers is plainly visible as the deep and more persistent spike in September 2008, preceded by a series of pronounced foreshocks over the course of the year.

Figure 9 shows stacked "ribbons" of daily data, using the same (red, blue and yellow) color to indicate relative levels (with black for missing data), where series are grouped vertically by instrument type.

[Insert Figure 9 about here]

Figures 9 and 10 illustrate a number of basic facts:

- equity markets and $VIX^{®}$ index responded strongly and immediately to the run on the repo, but WTI futures did not.

- $VIX^{®}$ liquidity remained relatively low throughout the crisis, but the liquidity of the equity market largely recovered with the exception of the financial and service sectors.

- liquidity in financial stocks remained depressed throughout late 2007 and 2008, consistent with increased uncertainty about the sector.

- liquidity implications of the Lehman Brothers failure were felt broadly for an extended period.

- hints of illiquidity foreshocks existed in some markets, including financial stocks (SIC 6) and certain bond sectors, that may ultimately help in crafting liquidity forecasts.

4.2 Explaining Liquidity Regimes

There appear to be strong relationships between changes in the level of liquidity and a number of macro variables, and while it is helpful to explore these relationships graphically the hierarchical model allows us to determine whether these relationships are statistically significant, particularly in the presence of other competing macro variables. We restrict our analysis of liquidity dynamics across multiple markets to the US Equity markets. We did this in part because of data consistency issues (there was no missing price-impact data for the U.S. Equity markets over the period of interest) and because these markets exhibit somewhat consistent behavior, see Figure 9. After visually exploring a range of potential macro variables, we selected the 11 macro variables describe in Table 2.

[Insert Table 2 about here]

We test the ability of the macro variables to recover the liquidity dynamics across these markets in two ways. First we calculate a hit rate, which is the proportion of the time that the Probit model, based solely on the macro variables, accurately predicts the state

identified by each of the underlying univariate models or we count the proportion of time that we accurately predict the state of D_{it} for each i and t using the current estimate of β, Σ and the macro data x_{it}. The naive hit rate is 33%, assuming random guessing, and the posterior average of Probit model's hit rate was 49% indicating that the macro variables are explaining a substantial portion of the liquidity dynamics. Second, we plotted the predicted probability of being in each state for each time point, using the Probit model, against the average probability of being in each state for each time point, see Figure 11. The way the predicted probabilities closely tracks the average probabilities confirm, again, the ability of the macro variables to explain the liquidity dynamics.

[Insert Figure 11 about here]

We standardized the macro variables (mean centered and divided by the standard deviation), so that the parameter estimates from the Probit portion of the hierarchical model, see Table 3, can be compared directly with respect to their size. Because of the way that we dealt with additive identification restriction (forcing the latent value for state 1 to always be zero), we get only get parameter estimates for state 2 and 3 (which are really the difference between the unrestricted parameters for each state and state 1). The negative intercepts indicate that state 1, the low price-impact or high-liquidity state, is the most prevalent state and the fact that the intercept for state 3 is more negative than for state 2 indicates that state 3, the low-liquidity state, is the least likely state.

[Insert Table 3 about here]

While all of the macro variables are statistically significant in some way (only the 3m Repo, Yield Curve and S&P 500 P/B Ratio are not significant in distinguishing the high-liquidity state from the mid-liquidity state), there are some interesting patterns to point out. First, there is a natural grouping among the macro variable with regards to

the pattern of the signs for the state 2 and 3 parameter estimates. The Dow Jones Real Estate Index has a negative, negative pattern, indicating that as this index climbs the probability of entering state 1, the high liquidity state, increases (a finding that we would expect given that our sample includes a bubble in the housing market). The next four (3m Repo, TED Spread, Break Even Inflation and Yield Curve) have a positive, negative pattern. This means as any of these rise, the probability of entering state 2 increases and as they fall the probability of entering either of the extreme states increases with an increased likelihood of entering state 3, the low-liquidity state. To help expand our understand of these parameter estimates, we can plot the macro variable versus the predicted probability for each state, see Figure 12 for the TED spread. The low level of liquidity in the equity markets following the bursting of the Internet bubble, coupled with the low TED spread results in the positive parameter value for state 2 (reflecting that there are periods where a low TED spread, which is negative because we mean centered the data, corresponds with a low-liquidity state, state 3), but the negative parameter value for state 3 comes about because of the spike in the TED during the crisis of 2008, which corresponds to another period of low-liquidity.

[Insert Figure 12 about here]

The next four (Moody's Baa Index, $VIX^{®}$, LIBOR and WTI) have a negative, positive pattern and are clear predictors of periods of low-liquidity. The $VIX^{®}$ has the strongest parameter estimate in absolute value and when the $VIX^{®}$ is high, the probability of being in state 2 drops and the probability of being in state 3 rises dramatically as indicated by plotting $VIX^{®}$ against the predicted probabilities in Figure 13.

[Insert Figure 13 about here]

The final macro variable (S&P 500 P/B Ratio) has a positive, positive pattern (although only the parameter for state 3 is statistically significant). This gives us the insight that

as the price of equities becomes large relative to the underlying book value of the firms, we tend to be in state 3, the low-liquidity state. One possible explanation for this is that a high price to book ratio reflects a potential asset bubble, which could lead investors to engage in herd behavior (e.g., piling into different individual stocks and driving up returns then pulling out suddenly causing large price drops), which could cause not only increased volatility but also larger price-impacts.

Clearly the hierarchical model allows us to obtain interesting insights into how macro variables relate to liquidity dynamics and offers a valuable tool for further investigating and understanding the drivers of liquidity across a wide range of markets.

5 Conclusion

Liquidity is an elusive, yet essential component of the modern financial system. It is elusive because conceptually it is hard to define, and empirically it is hard to measure and predict. More specifically, we attribute the challenges in liquidity measurement to three fundamental aspects of the phenomenon. Liquidity is latent, in the sense that the episodes of illiquidity we seek to understand are rare, and often emerge with little apparent warning. Liquidity is non-linear, in the sense that price impact does not respond proportionately to additional order flow, making it difficult to extrapolate from "ordinary" markets to the behavior of those markets under stress. Liquidty is endogenous, in the sense that it often emerges as a positive externality in very active markets, making those busy venues attractive to others who seek the assurance that counterparties will be available when needed.

We address the challenges of latency, non-linearity and endogeneity statistically with a Bayesian estimation of a hidden Markov chain individually for 33 separate time series covering the CRSP and TRACE universes of U.S. equities and corporate bonds, plus multiple expiries of two key futures contracts, the $VIX^{®}$ volatility contract and the WTI

oil contract. Three latent states (high, medium, and low price impact) are adequate to capture the observed liquidity structure of all 33 univariate series.

We also look for cross-sectional structure in the data by estimating a hierarchical Bayesian model, and testing the ability of several macroeconomic time series to recover the estimated aggregate liquidity dynamics. This exercise also permits an attribution of those estimated aggregate dynamics to meaningful economic interpretations. For reasons of data consistency, we have limited our initial efforts in this area to the U.S. equities markets.

Our results at this stage are very preliminary, but also very promising. In addition to testing for robustness and sensitivity, we see several immediate avenues for future research, including expanding the cross section of asset markets in the scope of analysis, comparing in more detail the liquidity behavior of wholesale funding markets, and experimenting wth alternative portfolio formation rules.

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Figures

Figure 1: The Leverage-Liquidity Cycle (Sources: Bloomberg L.P., OFR Analysis)

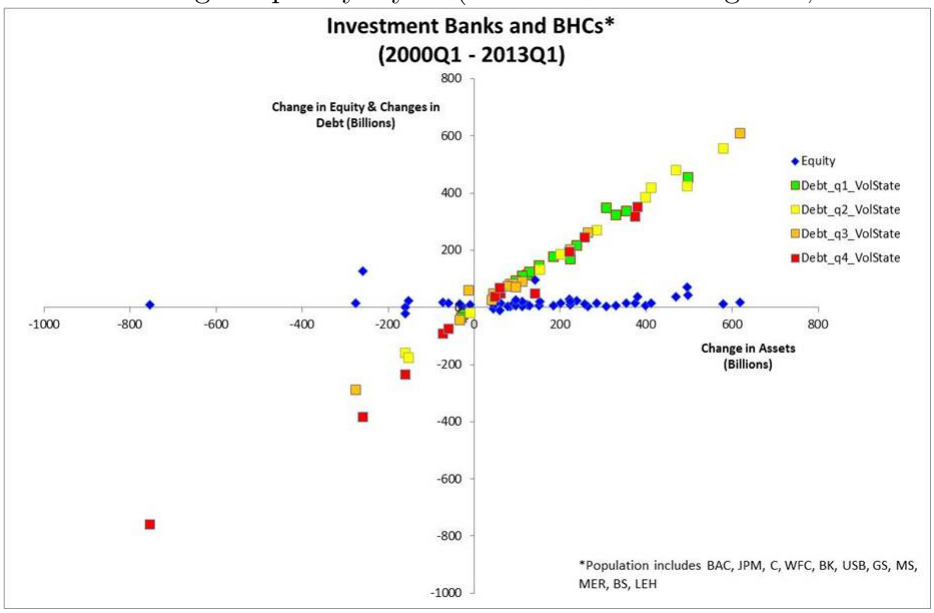


Figure 2: Liquidity Transformation via Financial Intermediation (Source: OFR Analysis)

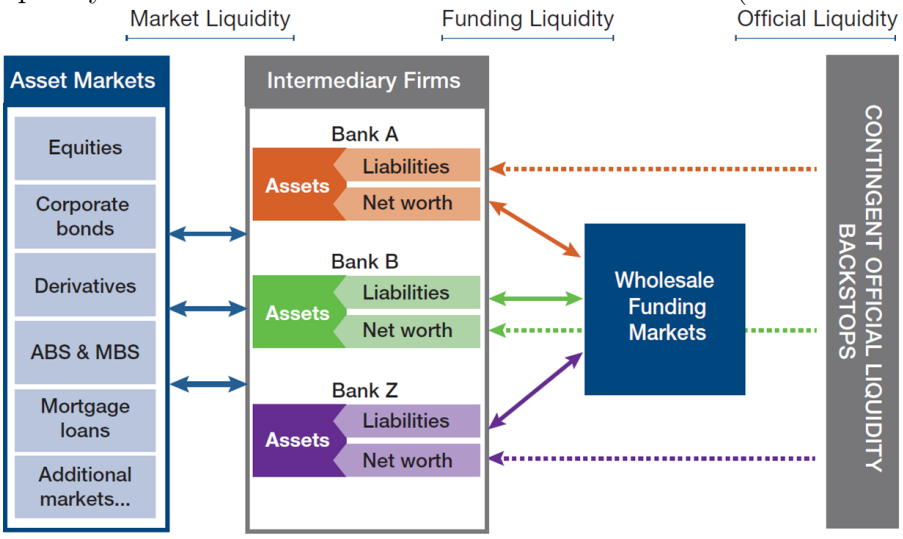


Figure 3: Bellwether Liquidity Measures from Wholesale Funding Markets (Source: Bloomberg)

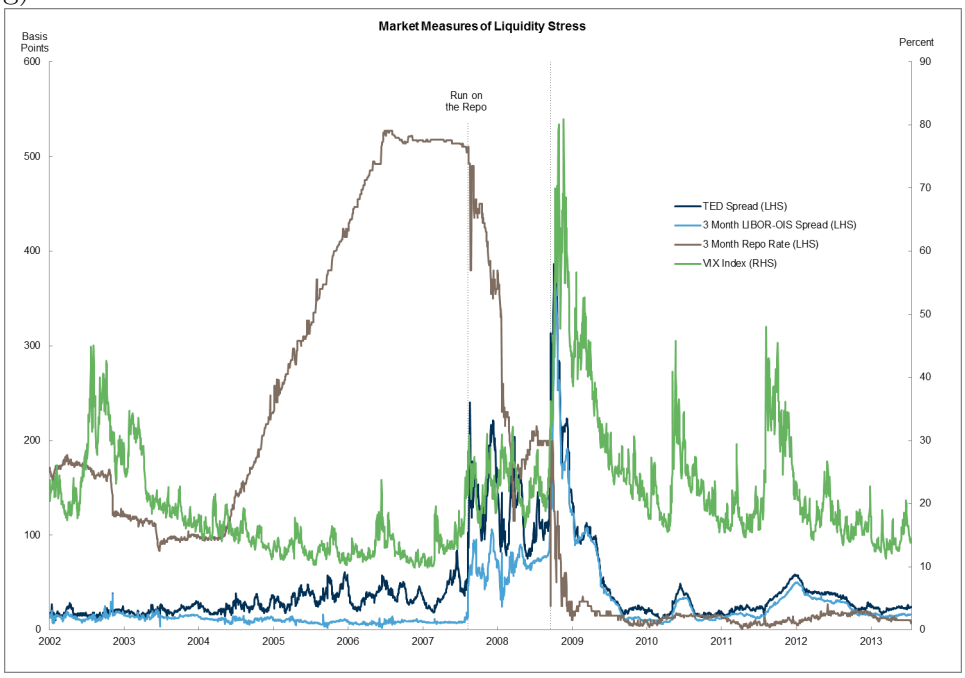


Figure 4: Bank Reserve Balances and Short-term Interest Rates (Source: St. Louis Federal Reserve Economic Data)

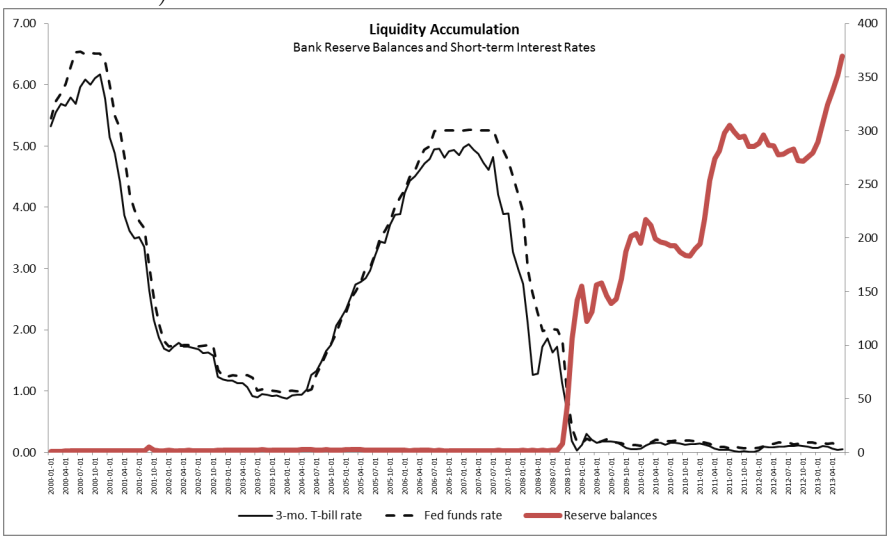
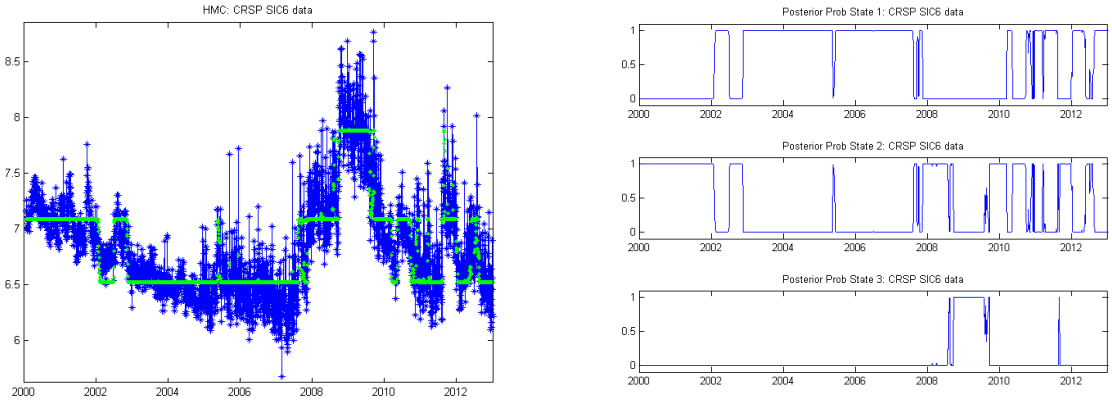


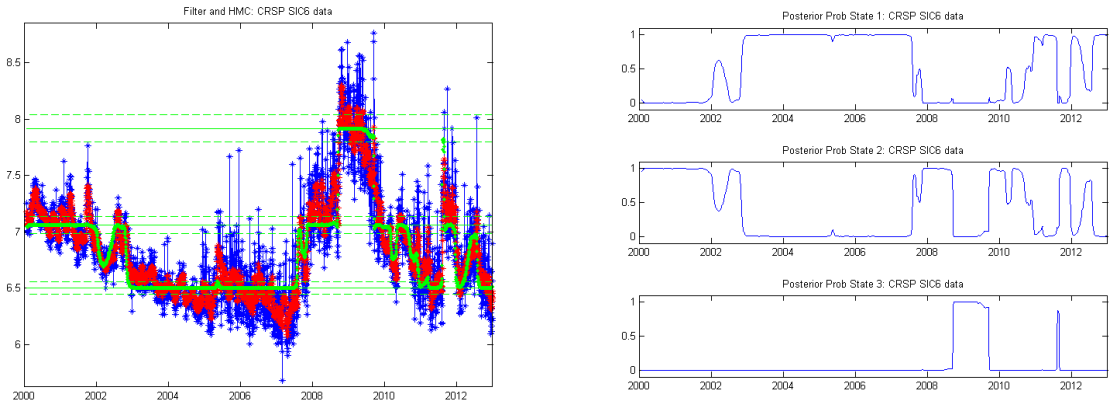
Figure 5: Equities, SIC6, Kyle-Obizhaeva Measure and HMC Estimates (Sources: Center for Research in Security Prices, Wharton Research Data Services, OFR Analysis)



(a) Kyle Measure & Level of Attraction

(b) State Probabilities

Figure 6: Equities, SIC6, Kyle-Obizhaeva Measure and MRHMC Estimates (Sources: Center for Research in Security Prices, Wharton Research Data Services, OFR Analysis)



(a) Kyle Measure, Mean Reverting & Level of Attraction

(b) State Probabilities

Figure 7: Daily Price Impact Range (Sources: Center for Research in Security Prices, Bloomberg L.P., Mergent, Inc., Wharton Research Data Services, Financial Industry Regulatory Authority, OFR Analysis)

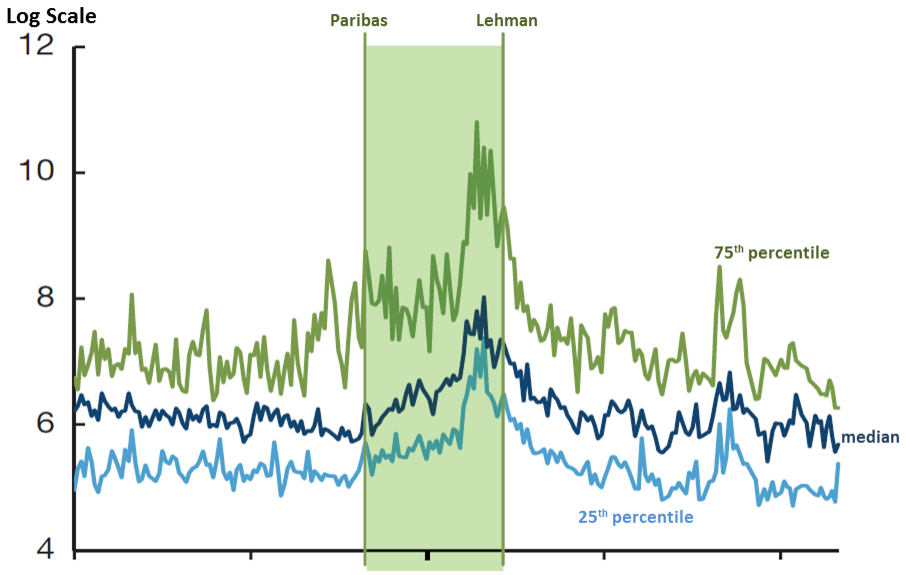


Figure 8: Probabilities of Liquidity States Across all 33 Markets (Sources: Center for Research in Security Prices, Bloomberg L.P., Mergent, Inc., Wharton Research Data Services, Financial Industry Regulatory Authority, OFR Analysis)

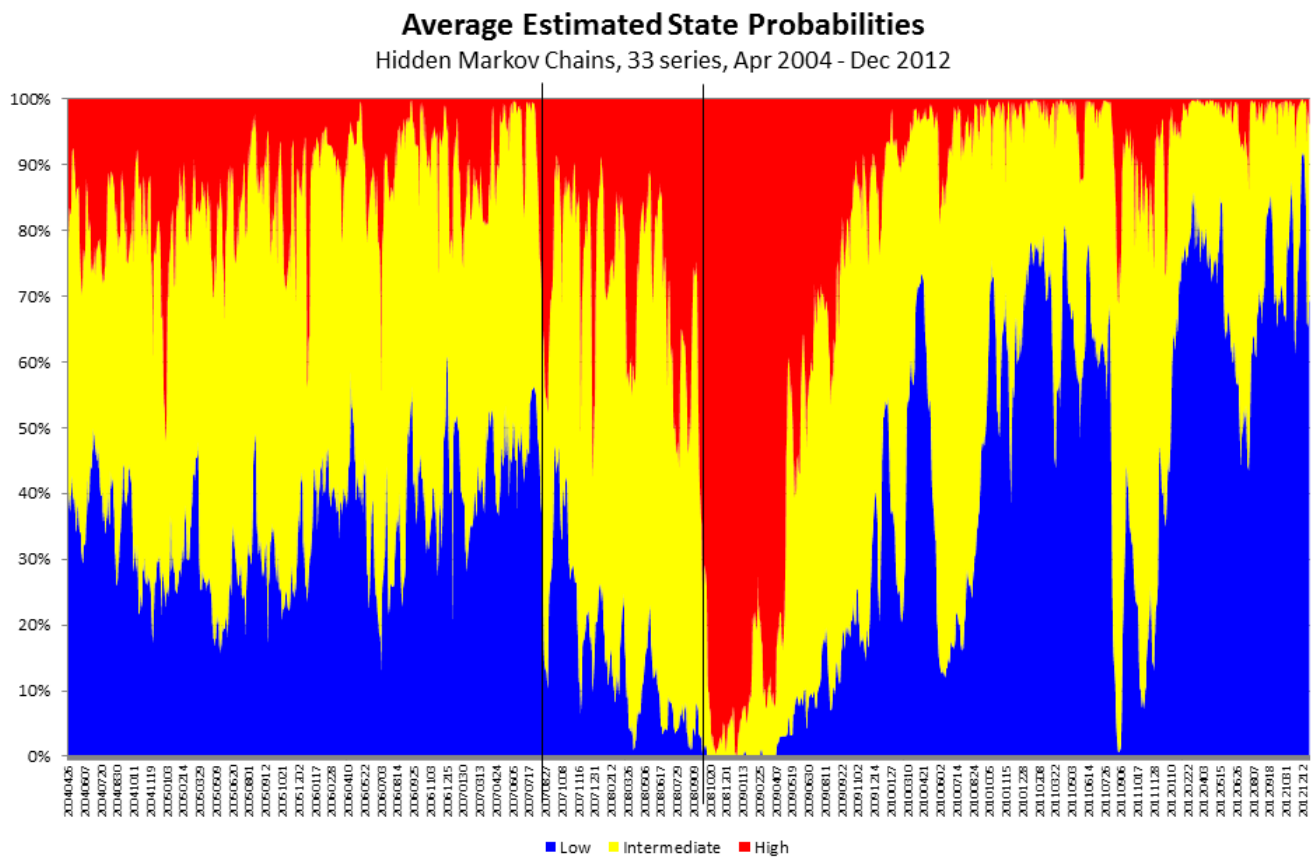


Figure 9: Daily Price-Impact Probabilities across all 33 Markets (Sources: Center for Research in Security Prices, Bloomberg L.P., Mergent, Inc., Wharton Research Data Services, Financial Industry Regulatory Authority, OFR Analysis)

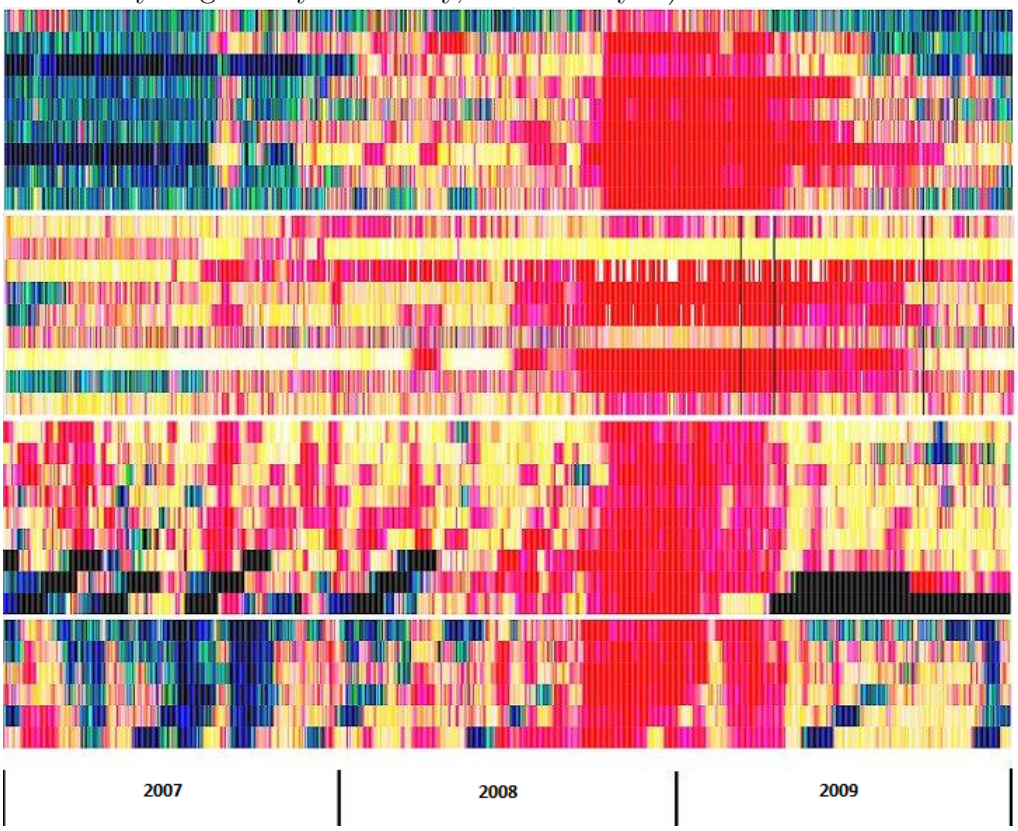


Figure 10: Daily Price-Impact Probabilities for Select Markets (Sources: Center for Research in Security Prices, Bloomberg L.P., Mergent, Inc., Wharton Research Data Services, Financial Industry Regulatory Authority, OFR Analysis)

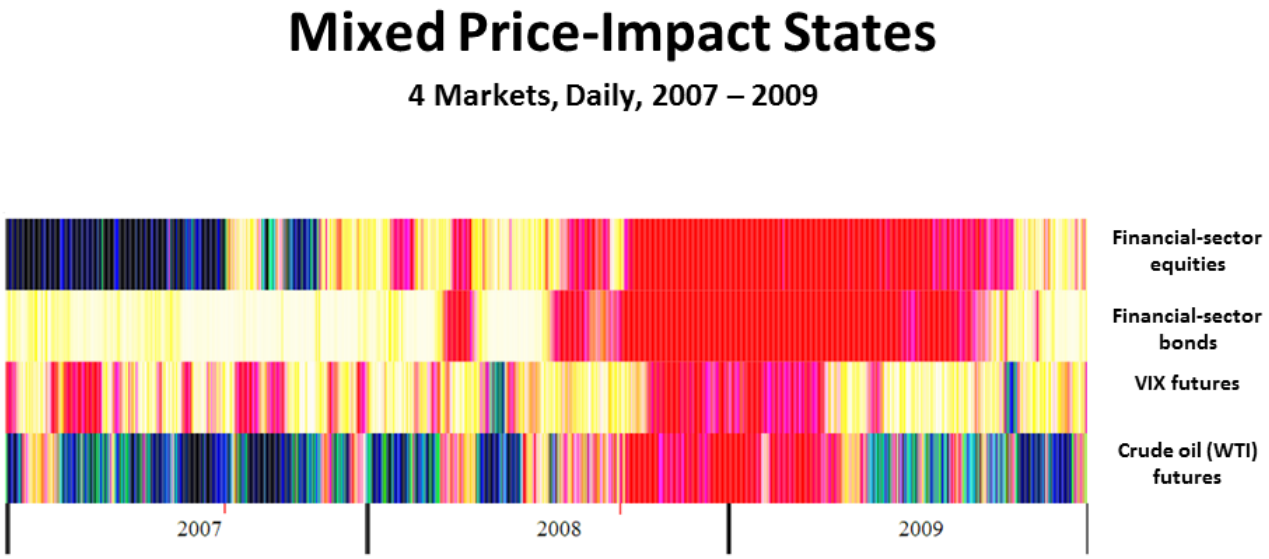


Figure 11: Average State Probabilities vs. Probit Predicted Probabilities (Sources: Center for Research in Security Prices, Wharton Research Data Services, Bloomberg L.P., OFR Analysis)

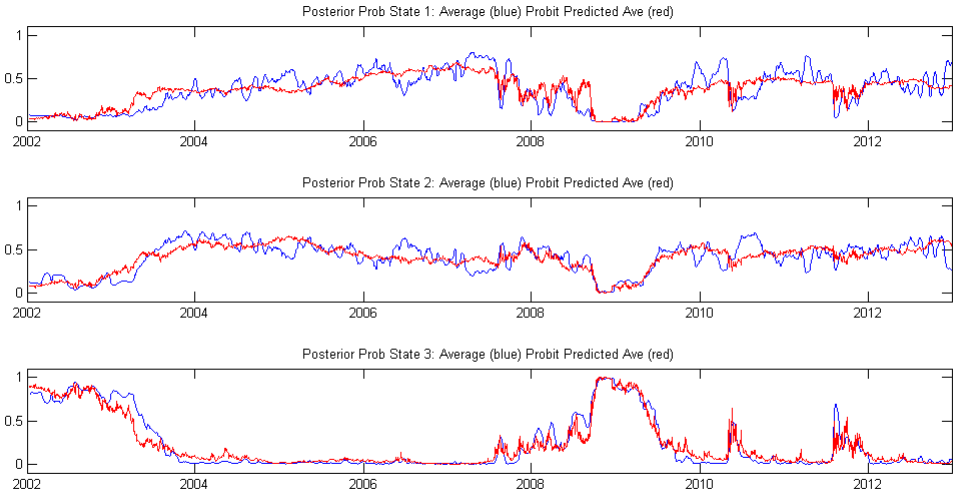


Figure 12: TED Spread and Average Probit Probabilities (Sources: Center for Research in Security Prices, Wharton Research Data Services, Bloomberg L.P., OFR Analysis)

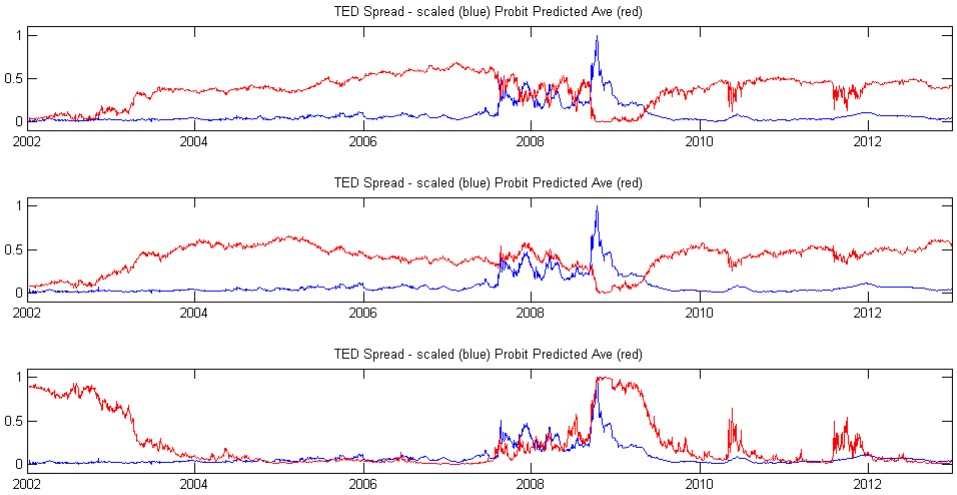
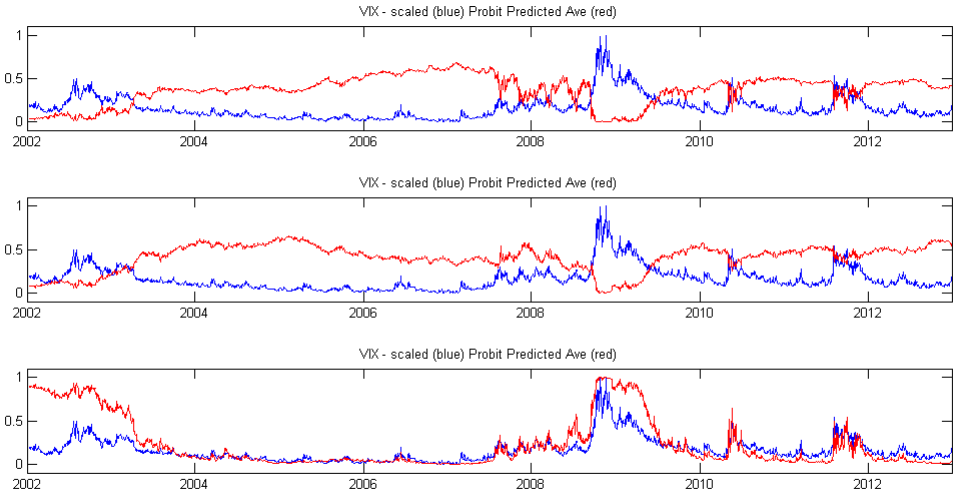


Figure 13: $VIX^{®}$ and Average Probit Probabilities (Sources: Center for Research in Security Prices, Wharton Research Data Services, Bloomberg L.P., OFR Analysis)



Tables

Table 1: Model Choice Results for all 33 Markets (Sources: Center for Research in Security Prices, Bloomberg L.P., Mergent, Inc., Wharton Research Data Services, Financial Industry Regulatory Authority, OFR Analysis)

	Posterior Probabilities			
Market Indexes	**Prob 2 States**	**Prob 3 States**	**Prob 4 States**	**Prob 5 States**
CRSP SIC0 Equity Index	**0.87367**	0.11973	0.0066	0
CRSP SIC1 Equity Index	0.3209	**0.3503**	0.3161	0.0127
CRSP SIC2 Equity Index	0.0000	**0.9021**	0.0948	0.0031
CRSP SIC3 Equity Index	0.1621	**0.7830**	0.0368	0.0181
CRSP SIC4 Equity Index	0.2672	**0.6753**	0.0558	0.0017
CRSP SIC5 Equity Index	0.0000	**0.6323**	0.3565	0.0113
CRSP SIC6 Equity Index	0.2066	**0.4797**	0.2933	0.0205
CRSP SIC7 Equity Index	0.2215	**0.5979**	0.1794	0.0012
CRSP SIC8 Equity Index	0.0014	**0.9022**	0.0728	0.0237
TRACE SIC0 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC1 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC2 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC3 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC4 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC5 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC6 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC7 Bond Index	TBD	TBD	TBD	TBD
TRACE SIC8 Bond Index	TBD	TBD	TBD	TBD
VIX MAT1 Index	0.00000	**0.62364**	0.37633	0.00000
VIX MAT2 Index	0.00191	**0.92470**	0.05590	0.01749
VIX MAT3 Index	0.00314	0.16869	**0.64321**	0.18496
VIX MAT4 Index	**0.60947**	0.36853	0.02200	0.00000
VIX MAT5 Index	0.06821	**0.82206**	0.07867	0.03106
VIX MAT6 Index	0.04602	**0.44739**	0.35430	0.15229
VIX MAT7 Index	0.14361	**0.66583**	0.16634	0.02422
VIX MAT8 Index	0.00000	**0.62618**	0.37337	0.00045
VIX MAT9 Index	0.00163	**0.65293**	0.31994	0.0255
WTI MAT1 Index	0.00000	**0.86957**	0.12249	0.00794
WTI MAT2 Index	0.00000	**0.79149**	0.12553	0.08298
WTI MAT3 Index	0.12943	**0.66173**	0.19548	0.01336
WTI MAT4 Index	0.00000	**0.82560**	0.17440	0.00000
WTI MAT5 Index	0.13576	**0.61316**	0.25108	0.00000
WTI MAT6 Index	0.00171	**0.79940**	0.19849	0.00040

Table 2: Macro Variables for Hierarchical (Multiple Market) Model

Macro Variable	Description
Three month Repo Rate	ICAP General Collateral Treasury 3-month repurchase agreement rate
Yield Curve	Yield on the constant maturity 10-year U.S. Treasury bond minus the yield on the constant maturity 2-year U.S. Treasury note
TED Spread	3-month LIBOR rate minus the 3-month U.S. Treasury bill yield
Moody's Baa Corporate Bond Index	Yield on the Moody's investment grade long-term corporate bond index
VIX® Index	Reflects the market estimate of future (30-day) volatility of the S&P 500
Dow Jones U.S. Real Estate Index	Index representing real estate investment trusts (REITs) and other companies investing directly or indirectly in real estate through development, management, or ownership
S&P 500 Price-to-book Ratio	Ratio of equity market value to book value per share of the S&P 500
Three-month LIBOR-OIS Spread	Difference between the 3-month LIBOR and the 3-month USD overnight index swap (OIS) rate
5-year U.S. Breakeven Inflation Rate	Calculated by subtracting the real yield of the 5-year inflation-linked maturity curve from the yield of the closest 5-year nominal Treasury maturity. The result is the market-implied inflation expectation over the next 5 years
WTI Front-Month Price	Futures price for the near-dated expiry of the WTI oil contract
DXY Dollar Index	Indicates the general international value of the USD, by averaging exchange of the USD against other major currencies

Table 3: Posterior Parameter Estimates Probit Portion of Hierarchical Model (Sources: Center for Research in Security Prices, Wharton Research Data Services, Bloomberg L.P., OFR Analysis)

	Posterior Mean		**Posterior StDev**		**t-Stat**	
Macro Variable	State 2	State 3	State 2	State 3	State 2	State 3
Intercept	**-0.21****	**-0.56****	0.01	0.01	-20.36	-43.27
Dow Jones Real Estate Index	**-0.29****	**-0.19****	0.03	0.01	-10.25	-20.69
3m Repo Rate	0.04	**-0.17****	0.06	0.05	0.68	-3.33
TED Spread	**0.27****	**-0.04***	0.05	0.01	5.82	-2.53
U.S. 5y Breakeven Inflation	**0.24****	**-0.13****	0.03	0.01	7.90	-9.35
Yield Curve (10y - 2y)	0.05	**-0.20****	0.04	0.04	1.22	-4.62
Moody's Baa Index	**-0.30****	**0.18****	0.04	0.03	-8.33	6.24
VIX®	**-0.16****	**0.43****	0.02	0.02	-7.68	22.98
LIBOR - OIS Spread	**-0.17****	**0.04***	0.04	0.02	-4.31	2.52
WTI	**-0.17****	**0.27****	0.03	0.01	-5.99	18.22
DXY Dollar Index	**-0.29****	**0.22****	0.03	0.02	-8.99	10.43
S&P 500 P/B Ratio	0.03	**0.27****	0.03	0.02	1.17	17.79

* Significant at a 95% confidence
** Significant at a 99% confidence

Table 4: Model Choice Exercise testing Delta Parameter for true state of 3 Liquidity Regimes

	Prob 2 States			**Prob 3 States**			**Prob 4 States**			**Prob 5 States**		
Delta	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3
0.001	0.9604	0.9603	0.9635	0.0396	0.0397	0.0365	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.005	0.8356	0.8125	0.8386	0.1627	0.1852	0.1596	0.0018	0.0023	0.0018	0.0000	0.0000	0.0000
0.01	0.7156	0.7346	0.7072	0.2773	0.2594	0.2859	0.0057	0.0050	0.0059	0.0015	0.0010	0.0011
0.05	0.0522	0.0810	0.0856	**0.8297**	**0.8026**	**0.7996**	0.0733	0.0758	0.0712	0.0448	0.0406	0.0436
0.1	0.0000	0.0000	0.0053	0.7756	0.7765	0.7750	0.1027	0.1026	0.0997	0.1217	0.1210	0.1201
0.15	0.0000	0.0000	0.0000	0.7049	0.7077	0.7136	0.1119	0.1086	0.1094	0.1832	0.1838	0.1770
0.2	0.0000	0.0000	0.0000	0.6481	0.6503	0.6451	0.1185	0.1158	0.1150	0.2335	0.2339	0.2399
0.25	0.0000	0.0000	0.0000	0.6055	0.5995	0.5958	0.1155	0.1153	0.1208	0.2789	0.2852	0.2834
0.3	0.0000	0.0000	0.0000	0.5563	0.5539	0.5536	0.1167	0.1224	0.1231	0.3270	0.3237	0.3234
0.35	0.0000	0.0000	0.0000	0.5151	0.5051	0.5143	0.1220	0.1240	0.1238	0.3630	0.3709	0.3619

	BIC 2 States			**BIC 3 States**			**BIC 4 States**			**BIC 5 States**		
Delta	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3	Data Set 1	Data Set 2	Data Set 3
0.001	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.005	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.01	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.05	**17570.71**	**17570.71**	**17570.71**	**17640.68**	**17640.68**	**17640.68**	**17732.26**	**17732.26**	**17732.26**	**17719.69**	**17719.69**	**17719.69**
0.1	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.15	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.2	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.25	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.3	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69
0.35	17570.71	17570.71	17570.71	17640.68	17640.68	17640.68	17732.26	17732.26	17732.26	17719.69	17719.69	17719.69